

Department of Mathematical Sciences
The Johns Hopkins University

STUDENT SEMINAR

Nevin Kapur
Department of Mathematical Sciences
The Johns Hopkins University

Monday, March 24, 2003
2 Shaffer Hall
4:00 p.m.

ADDITIVE FUNCTIONALS ON RANDOM SEARCH TREES

ABSTRACT

Search trees are fundamental data structures in computer science. Many important functionals of search trees satisfy recurrence relations of a simple additive form. These include the space requirement, internal path length, and the so-called shape functional. We derive asymptotics of moments and identify limiting distributions of such functionals under two commonly studied probability models—the random permutation model and the uniform (or Catalan) model.

For the random permutation model, our approach is based on establishing *transfer theorems* that link the order of growth of the input into a particular (deterministic) recurrence to the order of growth of the output. For the uniform model, our approach is based on the complex-analytic tool of *singularity analysis*. To facilitate a systematic analysis of these additive functionals we extend singularity analysis, a class of methods by which one can translate on a term-by-term basis an asymptotic expansion of a functional around its dominant singularity into a corresponding expansion for the Taylor coefficients of the function. The most important extension is the determination of how singularities are composed under the operation of Hadamard product of analytic power series.

The transfer theorems derived are used in conjunction with the method of moments to establish limit laws for m -ary search trees under the random permutation model. For the uniform model on binary search trees, the extended singularity analysis toolkit is employed to establish the asymptotic behavior of the moments of a wide class of functionals. These asymptotics are used, again in conjunction with the method of moments, to derive limit laws.