

Department of Mathematical Sciences  
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SEMINAR

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March 20, 2003  
304 Whitehead Hall  
Refreshments: 3:30 p.m.  
Seminar: 4:00 p.m.

PERCOLATION THRESHOLD BOUNDS, COUNTEREXAMPLES,  
AND CONJECTURES

ABSTRACT

One of the challenges in percolation theory is to understand the dependence of the critical probability on the structure of the underlying infinite graph. This survey talk will describe several recent results involving critical probability bounds, counterexamples, and conjectures: A complete study of the containment ordering of Archimedean and Laves lattices produced improved bounds for the percolation thresholds in several cases. Algorithmic improvements in the substitution method, using symmetry reduction, preferred partitions, and network flow algorithms, have produced very accurate bounds for the bond percolation thresholds of the Kagome lattice and  $(3, 12^2)$  lattice. The substitution method has improved bounds for both site and bond models for several other Archimedean lattices. Counterexamples to two long-held beliefs were found: (1) Two graphs with site and bond percolation thresholds in the opposite order were constructed. (2) Two graphs with percolation thresholds and average degrees in the same order were found, for both site and bond models.