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SEMINAR

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109 Maryland Hall
Refreshments: **10:30 a.m.**
Seminar: **11:00 a.m.**

EDGE WEIGHTS AND VERTEX COLORS

Abstract

A weighting of the edges of a graph with integer weights gives rise to a weighting of the vertices, the weight of a vertex being the sum of the weights of its incident edges. Such a weighting in turn induces a vertex coloring, by assigning the same color to vertices with the same weight.

An assignment of positive integer weights to the edges of a simple graph G is called *irregular* if the weighted degrees of the vertices are all different, i.e., the induced vertex coloring is trivial. The *irregularity strength* $s(G)$ is the maximal weight, minimized over all irregular assignments, and is set to ∞ if no such assignment is possible.

In the first part of my talk I will discuss results from a recent paper [1] where we show that $s(G) \leq c_1 n/\delta$ for graphs with maximum degree $\Delta \leq n^{1/2}$, and that $s(G) \leq c_2(\log n)n/\delta$ for graphs with $\Delta > n^{1/2}$, where c_1 and c_2 are explicit constants and δ denotes minimum degree of G.

It is natural to consider edge weightings where we require only that *adjacent* vertices have different weights—that is, that the vertex weighting induce a proper coloring of the graph. In this context, in [2] we raise the following question, in which a "nontrivial graph" refers to a connected graph with at least three vertices.

Question: *Is it possible to weight the edges of any nontrivial graph with the integers* $\{1, 2, 3\}$ *such that the resultant vertex weighting is a proper coloring?*

In the second part of my talk I will offer two pieces of information, given in [2], in relation to the above question: first, that a weighting *is* possible for 3-colorable graphs, and secondly that, if real, rather than just integer, weights are permitted, then a finite number of weights suffices for all graphs.

To prove all results in [1] and [2], we use a combination of deterministic and probabilistic techniques.

References

[1] R. J. Gould, A. Frieze, M. Karoński, and F. Pfender. On graph irregularity strength. *Journal of Graph Theory* **41** (2002), 120–137.

[2] M. Karoński, T. Łuczak, and A. Thomason. Edge weights and vertex colours. Submitted.