

Department of Mathematical Sciences
The Johns Hopkins University

SEMINAR

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November 21, 2002
304 Whitehead Hall
PRESEMINAR: 3:00 p.m.
Refreshments: 3:30 p.m.
Seminar: 4:00 p.m.

ON THE DIFFERENCE BETWEEN MEDIAN AND MEAN OF GAMMA
DISTRIBUTIONS AND A RELATED RAMANUJAN SEQUENCE

ABSTRACT

Let Y_μ be Poisson distributed with expectation μ and let λ_n be determined by $P(Y_{\lambda_n} \leq n) = 1/2$. Then λ_n is also the median of the distribution $\Gamma(n+1, 1)$.

What can be said about λ_n , or about $\alpha_n = \lambda_n - n$?

This problem is closely related to the following problems by Ramanujan:

1. Show that

$$\frac{e^n}{2} = 1 + n + \frac{n^2}{2} + \cdots + \frac{n^{n-1}}{(n-1)!} + \theta_n \cdot \frac{n^n}{n!},$$

where $1/3 < \theta_n \leq 1/2$ for $n = 0, 1, 2, \dots$

2. Show that

$$\theta_n = \frac{1}{3} + \frac{4}{135(n + k_n)},$$

where $2/21 < k_n \leq 8/45$.