Unified Approach for Minimizing Composite Norms

In this talk, I will describe a first-order augmented Lagrangian algorithm (FALC) [1] to solve

\[
\min_{X \in \mathbb{R}^{m \times n}} \quad \mu_1 \|\sigma(X)\|_\alpha + \mu_2 \|C(X) - d\|_\beta + \langle R, X \rangle \\
\text{subject to} \quad G(X) = h, \quad F(X) \preceq G, \quad \|A(X) - b\|_\gamma \leq \rho,
\]

where \(\sigma : \mathbb{R}^{m \times n} \to \mathbb{R}^{\min\{m,n\}}\) is a function returning singular values of its argument, the matrix norm \(\|\sigma(\cdot)\|_\alpha\) denotes either the Frobenius, the nuclear, or the \(\ell_2\)-operator norm, the vector norms \(\|\cdot\|_\beta\) and \(\|\cdot\|_\gamma\) denote either the \(\ell_1\)-norm, \(\ell_2\)-norm or the \(\ell_\infty\)-norm; and \(A(\cdot), C(\cdot), F(\cdot)\) and \(G(\cdot)\) are linear operators. Basis Pursuit, Matrix Completion, Robust PCA, Stable Principle Component Pursuit and even SDP problems are special cases of this formulation. The methodology proposed in this paper can handle problems with fairly general feasible sets defined by linear equalities, conic constraints and norm-ball type inequalities. FALC converges to an optimal solution \(X^*\) of the problem (1) and for all small enough \(\epsilon > 0\), the iterates \(X_k\) computed by FALC are \(\epsilon\)-feasible and \(\epsilon\)-optimal after \(O(\log(1/\epsilon))\) iterations, which requires \(O(\frac{1}{\epsilon})\) operations in total where the complexity of each operation is dominated by computing a singular value decomposition. This is a joint work with Garud Iyengar.

References