1.

**Problem 2.3.7 p 152 re curved EF (see also p 57 and p 125)**

Let \( F = \{ f_\lambda(x) = \lambda e^{-\lambda x} : \lambda \in \mathbb{R}_+, x \in \mathbb{R}_+ \} \).

Let \( X_i \text{ ind } f_{\lambda_i} \in F, i = 1, 2 \).

Assume \( \lambda_i = e^{i\theta} \) for \( \theta \in \mathbb{R} \).

(a) Argue existence properties of \( \hat{\theta}_{MLE}(X_1, X_2) \).

(b) Argue uniqueness or non-uniqueness of \( \hat{\theta}_{MLE}(X_1, X_2) \).

(c) Compare with the case \( \lambda_i = i\theta, \theta \in \mathbb{R}_+ \).

**suggested grading:**

(a) 1 point for “mle existence guaranteed in curved EF from Theorem 2.3.3”;
(b) 1 point for “mle uniqueness NOT guaranteed in curved EF”;
(c) 1 point for “mle existence and uniqueness guaranteed in EF from Theorem 2.3.1”.

2.

**Problem 2.4.12 p 157 re EM justification equation (2.4.8) p 134**

Let \( X \sim p \in \{ p_\theta : \theta \in \Theta \} \) and \( S(X) \sim q \in \{ q_\theta : \theta \in \Theta \} \),

where \( X \) is “complete data” and \( S(X) \) is “observed data” as in Section 2.4.4.

Assume that \( |\text{range}(X)| < \infty \).

Show that \( E_{\theta_0}[p_\theta(X)/p_{\theta_0}(X)|S(X) = s] = q_\theta(s)/q_{\theta_0}(s) \).

**suggested grading:** 3 points, all or nothing.

3.

**Theorem 5.2.2(i) pp 303-304 re pp 122-123**

Let \( F = \{ f_\eta : \eta \in \mathcal{E} \} \) be a canonical exponential family with \( \dim(F) = \text{rank}(F) = 1 \) and \( \mathcal{E} = \mathcal{E}^o \).

Let \( X_1, \ldots, X_n \text{ id } f \in F \).

Show that \( P[\arg\max_{\eta \in \mathcal{E}} \prod_{i=1}^n f_{\eta}(X_i) = 1] \to 1 \) as \( n \to \infty \).

**suggested grading:**

1 point for “if \( T \) has a probability density function, then existence & uniqueness almost surely from Theorem 2.3.2”;

1 point for “if \( T \) has a probability mass function, then Theorem 2.3.2 does not apply”;

1 point for “if \( T \) has a probability mass function, then LIMITING existence & uniqueness almost surely from Theorem 5.2.2(i)”.

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