OPTIMIZATION ALGORITHMS (550.662)
Homework 9, Spring 2009
Due Friday, April 10, 2009

Problem 1

Let $f(\xi_1, \xi_2) = 4\xi_1^2 - 4\xi_1\xi_2 + 2\xi_2^2$, and let $x_0 = (2, 3)$. Carry out three iterations of the steepest descent algorithm with exact line-search to find x_3.

Problem 2

Let $\{x^k\}$ be generated by the steepest descent method with an exact line-search. Show that $(x_{k+2} - x_{k+1})^T (x_{k+1} - x_k) = 0$.

Problem 3

(a) Find the quadratic approximation $q(x)$ of

$$f(x) = 8\xi_1^2 + 8\xi_2^2 - \xi_1^4 - \xi_2^4 - 1$$

at $\hat{x} = (\frac{1}{2}, \frac{1}{2})$. That is

$$q(x) = f(\hat{x}) + \nabla f(\hat{x})^T (x - \hat{x}) + \frac{1}{2}(x - \hat{x})^T \nabla^2 f(\hat{x})(x - \hat{x}).$$

(b) Find the minimizer of q.

Problem 4

Let f be a twice continuously differentiable function. Show that for sufficiently large μ, the direction

$$d = -(\nabla^2 f(x) + \mu I)^{-1} \nabla f(x)$$

is a descent direction.

Problem 5

Let B be a $n \times n$ positive definite matrix and define a vector norm in R^n by: $||x||_B = \sqrt{x^T B x}$. The steepest descent direction with respect to the norm $|| \cdot ||_B$ is the vector which solves:

$$\begin{array}{ll} \min & \nabla f(x)^T d \\ \text{s.t.} & ||d||_B = 1. \end{array}$$

Show that the steepest direction is in the direction $-B^{-1}\nabla f(x)$. (Hint: Let $B = LL^T$ and consider the transformation $z = L^T d$.)