Problem 1
Let \( f(\xi_1, \xi_2) = 4\xi_1^2 - 4\xi_1\xi_2 + 2\xi_2^2 \), and let \( x_0 = (2, 3) \). Carry out three iterations of the steepest descent algorithm with exact line-search to find \( x_3 \).

Problem 2
Let \( \{x^k\} \) be generated by the steepest descent method with an exact line-search. Show that

\[
(x_{k+2} - x_{k+1})^T(x_{k+1} - x_k) = 0.
\]

Problem 3
(a) Find the quadratic approximation \( q(x) \) of

\[
f(x) = 8\xi_1^2 + 8\xi_2^2 - \xi_1^4 - \xi_2^4 - 1
\]

at \( \hat{x} = (\frac{1}{2}, \frac{1}{2}) \). That is

\[
q(x) = f(\hat{x}) + \nabla f(\hat{x})^T(x - \hat{x}) + \frac{1}{2}(x - \hat{x})^T\nabla^2 f(\hat{x})(x - \hat{x}).
\]

(b) Find the minimizer of \( q \).

Problem 4
Let \( f \) be a twice continuously differentiable function. Show that for sufficiently large \( \mu \), the direction

\[
d = -(\nabla^2 f(x) + \mu I)^{-1}\nabla f(x)
\]

is a descent direction.

Problem 5
Let \( B \) be a \( n \times n \) positive definite matrix and define a vector norm in \( \mathbb{R}^n \) by:

\[
\|x\|_B = \sqrt{x^T B x}.
\]

The steepest descent direction with respect to the norm \( \| \cdot \|_B \) is the vector which solves:

\[
\min \quad \nabla f(x)^T d
\]

s.t. \( \|d\|_B = 1 \).

Show that the steepest direction is in the direction \(-B^{-1}\nabla f(x)\). (Hint: Let \( B = LL^T \) and consider the transformation \( z = L^T d \).)