Problem 1
Let
\[ f(\xi_1, \xi_2) = (\xi_1 - 2)^4 + (\xi_1 - 2)^2\xi_2^2 + (\xi_2 + 1)^2. \]

(a) Show that \( f \) has a local minimizer at \( x^* = (2, -1) \).

(b) Carry out one step of the BFGS method with stepsize \( \lambda = 1 \) from \( x_0 = (1, 1) \) and \( B_0 = \nabla^2 f(x^0) \). Compute both \( x_1 \) and \( B_1 \).

Problem 2
Consider the nonlinear least-squares problem:
\[ \min(\xi_1^2 - 2\xi_2 + 1)^2 + (2\xi_1\xi_2 - 2)^2 + (\xi_1 - \xi_2)^2. \]

(a) Carry out one step of the Gauss-Newton method with stepsize \( \lambda = 1 \) from \( x_0 = (0, 0) \).

(b) Carry out one step of the Newton method with stepsize \( \lambda = 1 \) from \( x_0 = (0, 0) \).

Problem 3
Let \( \tilde{B} \) be obtained from a positive definite matrix \( B \) and nonzero vectors \( s \) and \( y \) by using the BFGS formula. Show that if \( \tilde{B} \) is positive definite then \( y^T s > 0 \).

Problem 4
Comment: This problem is on the product form of the BFGS formula which is useful for a conjugate direction approach for solving nonlinear problems.

Let \( \tilde{B} \) be obtained from \( B, y, \) and \( s \) (with \( y^T s \neq 0 \)) from the BFGS formula. Let
\[ A = \left( I - \frac{sy^T}{y^T s} + \frac{1}{\sqrt{s^T B sy s}} s s^T B \right). \]

Show that \( A^T \tilde{B} A = B \).