OPTIMIZATION ALGORITHMS (550.662)
Homework 12, Spring 2009
Due Friday, May 1, 2009

Problem 1

Consider the following nonlinear program:

$$\begin{aligned} \min \quad & f(x) \\ s.t. \quad & g_i(x) \leq 0 \quad i = 1, \cdots, m \\ & h_j(x) = 0 \quad j = 1, \cdots, p \end{aligned}$$

where all functions are differentiable. Let $\hat{x}$ be a point in R^n. Show that if $d = 0$ is a K-K-T point of the following quadratic program then $\hat{x}$ is a K-K-T point of the above nonlinear program:

$$\begin{aligned} \min \quad & \nabla f(\hat{x})^T d + d^T Q d \\ s.t. \quad & g_i((\hat{x}) + \nabla g_i(\hat{x})^T d \leq 0 \quad i = 1, \cdots, m \\ & h_j(\hat{x}) + \nabla h_j(\hat{x})^T d = 0 \quad j = 1, \cdots, p \end{aligned}$$

where Q is a positive definite matrix.

Problem 2

Use the gradient projection method to solve the following problem with $(0, 0)$ as the starting point.

$$\begin{aligned} \min \quad & (\xi_1 - 1)^2 + (\xi_2 - 2)^2 \\ s.t. \quad & 0 \leq \xi_1 \leq 1 \\ & 0 \leq \xi_2 \leq 1. \end{aligned}$$

Problem 3

Consider the following quadratic program:

$$\begin{aligned} \min \quad & \tfrac{1}{2} x^T Q x - c^T x \\ s.t. \quad & Ax = b \end{aligned}$$

where Q is a positive definite matrix and A is a matrix with linearly independent rows. Let x_α be the minimum point of the the penalty function:

$$P_\alpha(x) = x^T Q x - c^T x + \frac{\alpha}{2}(Ax - b)^T (Ax - b).$$

Show that when $\alpha \to \infty$, $\{x_\alpha\}$ converges to the solution of the quadratic program.