Problem 1
Consider the following nonlinear program:

\[
\begin{align*}
\text{min} & \quad f(x) \\
\text{s.t.} & \quad g_i(x) \leq 0 \quad i = 1, \ldots, m \\
& \quad h_j(x) = 0 \quad j = 1, \ldots, p
\end{align*}
\]

where all functions are differentiable. Let \( \hat{x} \) be a point in \( \mathbb{R}^n \). Show that if \( d = 0 \) is a K-K-T point of the following quadratic program then \( \hat{x} \) is a K-K-T point of the above nonlinear program:

\[
\begin{align*}
\text{min} & \quad \nabla f(\hat{x})^T d + d^T Q d \\
\text{s.t.} & \quad g_i(\hat{x}) + \nabla g_i(\hat{x})^T d \leq 0 \quad i = 1, \ldots, m \\
& \quad h_j(\hat{x}) + \nabla h_j(\hat{x})^T d = 0 \quad j = 1, \ldots, p
\end{align*}
\]

where \( Q \) is a positive definite matrix.

Problem 2
Use the gradient projection method to solve the following problem with \((0, 0)\) as the starting point.

\[
\begin{align*}
\text{min} & \quad (\xi_1 - 1)^2 + (\xi_2 - 2)^2 \\
\text{s.t.} & \quad 0 \leq \xi_1 \leq 1 \\
& \quad 0 \leq \xi_2 \leq 1.
\end{align*}
\]

Problem 3
Consider the following quadratic program:

\[
\begin{align*}
\text{min} & \quad \frac{1}{2} x^T Q x - c^T x \\
\text{s.t.} & \quad Ax = b
\end{align*}
\]

where \( Q \) is a positive definite matrix and \( A \) is a matrix with linearly independent rows. Let \( x_\alpha \) be the minimum point of the the penalty function:

\[
P_\alpha(x) = x^T Q x - c^T x + \frac{\alpha}{2} (Ax - b)^T (Ax - b).
\]

Show that when \( \alpha \to \infty \), \( \{x_\alpha\} \) converges to the solution of the quadratic program.