Special thanks to Aubin for providing the excellent answer to certain problems

Prob. 1

Suppose that x_i is the departing basic variable. Let c_i be the reduced costs for x_i. This means that we must have $c_i < 0$ because we can only pivot on a variable with negative reduced cost. Then the tableau corresponding to this vertex has the following form.

	x_i
	c_i
β	α

After we pivot on the α, the tableau becomes

	x_i
	0
β/α	1

The reduced cost for x_i after pivoting is zero. Since we only pivot on variables with negative reduced costs, we cannot again pivot on x_i. That is, it cannot be an entering variable immediately after being a leaving variable.

Prob. 4

$$\begin{array}{rl} \min & 3x_1 - 4x_2 - x_3 - 2x_4 - 3x_5 \\ \text{subject to} & x_1 + x_2 + x_3 - x_4 + 2x_5 \leq 12 \\ & x_1 - 2x_2 - x_3 - x_4 - x_5 \geq -30 \\ & x_i \geq 0, \quad i = 1,2,3,4,5. \end{array}$$

We start with the initial tableau. We will introduce two slack variables to convert this to standard form. We will call these slack variables u_1 and u_2. We will not explicity write in the column corresponding to the z variable, where $z = -c^\top x$.

$$T_1 = \left[\begin{array}{c|ccccccc} 0 & 3 & -4 & -1 & -2 & -3 & 0 & 0 \\ \hline 12 & 1 & 1 & 1 & -1 & 2 & 1 & 0 \\ 30 & -1 & 2 & 1 & 1 & 1 & 0 & 1 \end{array}\right]$$

The basic variables are, in order, (z, u_1, u_2).

$$B_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Using Bland's rule, we pivot on x_2. By the ratio test, we will pivot on the first row, on the 1 in that column.

$$E_1 = \begin{bmatrix} 1 & 4 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

After this pivot, the basic variables are (z, x_2, u_2).

$$B_2^{-1} = E_1 B_1^{-1} = \begin{bmatrix} 1 & 4 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$$T_2 = B_2^{-1} T_1 = \left[\begin{array}{c|ccccc} 48 & 7 & 0 & 3 & -6 & \\ \hline 12 & 1 & 1 & 1 & -1 & \cdots \\ 6 & -3 & 0 & -1 & 3 & \end{array}\right]$$

We calculate T_2 until we find the first column with negative reduced costs. So we pivot on x_4. Since there is only one positive number in that column, we will pivot on the last row. After this pivot, the basic variables will be (z, x_2, x_4)

$$E_2 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1/3 \\ 0 & 0 & 1/3 \end{bmatrix}$$

$$B_3^{-1} = E_2 B_2^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1/3 & 1/3 \\ 0 & -2/3 & 1/3 \end{bmatrix}$$

$$T_3 = B_3^{-1} T_1 = \left[\begin{array}{r|rrrrrr} 60 & 1 & 0 & 1 & 0 & -1 & \\ \hline 14 & 0 & 1 & 2/3 & 0 & 1 & \cdots \\ 2 & -1 & 0 & -1/3 & 1 & -1 & \end{array}\right]$$

So now we pivot on x_5. Again, there is only one choice of pivot.

$$E_3 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$B_4^{-1} = E_3 B_3^{-1} \begin{bmatrix} 1 & 1/3 & 7/3 \\ 0 & 1/3 & 1/3 \\ 0 & -1/3 & 2/3 \end{bmatrix}$$

$$T_4 = B_4^{-1} T_1 \left[\begin{array}{r|rrrrrrr} 74 & 1 & 1 & 5/3 & 0 & 0 & 1/3 & 7/3 \\ \hline 14 & 0 & 1 & 2/3 & 0 & 1 & 1/3 & 1/3 \\ 16 & -1 & 1 & 1/3 & 1 & 0 & -1/3 & 2/3 \end{array}\right]$$

All the reduced costs are nonnegative. We are at an end. We read off the optimal solution.

$$\hat{x} = (0, 0, 0, 16, 14) \quad c^\top \hat{x} = -74.$$