Prob 1

Exercise 10.8
We assume that the graph is complete. Let x_e be equal to 1 if the edge e is used in any of the routes, 0 otherwise. The formulation is as follows.

$$\begin{aligned}
\text{minimize} \quad & \sum_{e \in \mathcal{E}} d_e x_e \\
\text{subject to} \quad & \sum_{e \in \delta(i)} x_e = 2, && i \neq 1, \\
& \sum_{e \in \delta(1)} x_e = 2m, \\
& \sum_{e \in \delta(S)} x_e \geq 2\left\lceil \frac{\sum_{i \in S} b_i}{Q} \right\rceil, && S \subset \mathcal{N} \setminus \{1\},\ S \neq \emptyset, \\
& x_e \in \{0,1\}.
\end{aligned}$$

Prob 2

Exercise 10.10
Let $\mathcal{T} = \{0, 1, \ldots, T\}$, where T is an upper bound on the total time to complete all jobs. Each job consists of m phases denoted by (j, r), $r = 1, \ldots, m$. The rth phase of job j is processed by machine $M_{j(r)}$. Let $J = \{(j,r) \mid j = 1, \ldots, n,\ r = 1, \ldots K\}$. The processing time of the rth phase of job j

is then $p_{j(r),j}$. Let $x_{j,r,t}$ be equal to 1, if the rth phase of job j is being processed at time t, and 0 otherwise. Let $C_{j,r}$ be the completion time of the rth phase of job j. Then, the objective is

$$\text{minimize} \quad \sum_j C_{j,m}.$$

Clearly, the total time phase r of job j spends in the system equals its processing time, i.e.,

$$\sum_{t \in \mathcal{T}} x_{j,r,t} = p_{j(r),j}.$$

In addition,

$$\begin{aligned}
\sum_{t \in \mathcal{T}} t x_{j,r,t} &= \sum_{t = C_{j,r} - p_{j(r),j}}^{C_{j,r}} t \\
&= C_{j,r}\left(1 + p_{j(r),j}\right) - \frac{p_{j(r),j}\left(1 + p_{j(r),j}\right)}{2}.
\end{aligned}$$

The precedence constraints are modeled as follows:

$$C_{j,r} \geq C_{j,r-1} + p_{j(r),j}.$$

Let $M_k = \{(j,r) \mid \text{the } r\text{th face of job } j \text{ is processed by machine } k\}$. Because each machine should process at most one job at every time

$$\sum_{(j,r) \in M_k} x_{j,r,t} \leq 1, \quad \forall\, t \in \mathcal{T}.$$

3) $x_{ij} = \begin{cases} 1 & \text{if travel from city } i \text{ to } j \\ 0 & \text{o.w.} \end{cases}$

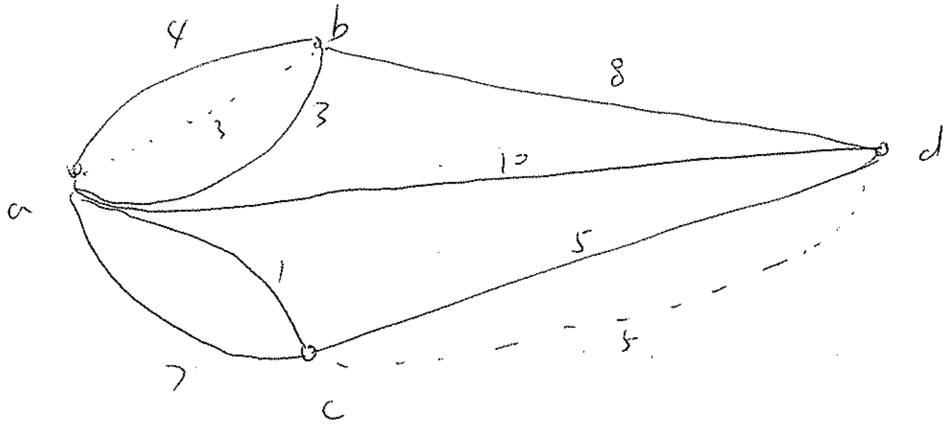
$$\min \sum_{i=1}^{5} \sum_{j=1}^{5} c_{ij} x_{ij}$$

s.t. $\sum_{\substack{j=1 \\ j \neq i}}^{5} x_{ij} = 1 \quad i = 1, 2, \cdots 5$

$\sum_{\substack{j=1 \\ j \neq i}}^{5} x_{ji} = 1. \quad i = 1, 2, \cdots 5$

$\sum_{\substack{i \in S \\ j \notin S}} x_{ij} \geq 1 \quad$ for any $S \subseteq \{1, 2, 3, 4, 5\} \quad 2 \leq |S| \leq 3$

4) Every node has odd degree. so we could use coupling.



optimal path. a b a b d a c d c a

optimal cost. 46