Prob 1

Exercise 10.8
We assume that the graph is complete. Let \( x_e \) be equal to 1 if the edge \( e \) is used in any of the routes, 0 otherwise. The formulation is as follows.

\[
\begin{align*}
\text{minimize} & \quad \sum_{e \in E} d_e x_e \\
\text{subject to} & \quad \sum_{e \in E(i)} x_e = 2, & i \neq 1, \\
& \quad \sum_{e \in E(1)} x_e = 2m, \\
& \quad \sum_{e \in E(S)} x_e \geq 2 \left( \sum_{i \in S} x_i \right) & S \subseteq N \setminus \{1\}, S \neq \emptyset, \\
& & x_e \in \{0, 1\}.
\end{align*}
\]

Prob 2

Exercise 10.10
Let \( T = \{0, 1, \ldots, T\} \), where \( T \) is an upper bound on the total time to complete all jobs. Each job consists of \( m \) phases denoted by \((j, r), r = 1, \ldots, m\). The \( r \)th phase of job \( j \) is processed by machine \( M_{j,r} \). Let \( J = \{(j, r) \mid j = 1, \ldots, n, r = 1, \ldots, R\} \). The processing time of the \( r \)th phase of job \( j \) is then \( p_{j,r} \). Let \( x_{j,r,t} \) be equal to 1, if the \( r \)th phase of job \( j \) is being processed at time \( t \), and 0 otherwise. Let \( C_{j,r} \) be the completion time of the \( r \)th phase of job \( j \). Then, the objective is

\[
\text{minimize} \quad \sum_{j} C_{j,m}.
\]

Clearly, the total time phase \( r \) of job \( j \) spends in the system equals its processing time, i.e.,

\[
\sum_{t \in T} x_{j,r,t} = p_{j,r}.
\]

In addition,

\[
\sum_{t \in T} \sum_{r} x_{j,r,t} = \sum_{t = 0}^{C_{j,r} - p_{j(r),s}} t = C_{j,r}(1 + p_{j(r),s}) - \frac{p_{j(r),s}(1 + p_{j(r),s})}{2}.
\]

The precedence constraints are modeled as follows:

\[
C_{j,r} \geq C_{j,r-1} + p_{j(r),s}.
\]

Let \( M_k = \{(j, r) \mid \text{the } r\text{th face of job } j \text{ is processed by machine } k\} \). Because each machine should process at most one job at every time

\[
\sum_{(j, r) \in M_k} x_{j,r,t} \leq 1, \quad \forall t \in T.
\]
3) \( \chi_{ij} = \begin{cases} 1 & \text{if travel from city } i \text{ to } j \\ 0, \text{ otherwise} \end{cases} \)

\[
\min \left\{ \sum_{i=1}^{5} \frac{z}{2} \chi_{ij} \right\} \\
\text{s.t.} \quad \sum_{j \neq i} \chi_{ij} = 1, \quad i = 1, 2, \ldots, 5 \\
\sum_{i \neq j} \chi_{ij} = 1, \quad j = 1, 2, \ldots, 5 \\
\sum_{s \in \{1, 2, 3, 4\}} \sum_{i \neq s} |\chi_{ij}| = 2 \quad \text{for any } s \in \{1, 2, 3, 4\}, \quad 2 \leq |s| \leq 3
\]

4) Every node has odd degree, so we could use coupling.

Optimal path: a b a b d a c d e a

Optimal cost: 46