

Prob 1

Exercise 10.8

We assume that the graph is complete. Let x_e be equal to 1 if the edge e is used in any of the routes, 0 otherwise. The formulation is as follows.

$$\begin{aligned}
 & \text{minimize} && \sum_{e \in \mathcal{E}} d_e x_e \\
 & \text{subject to} && \sum_{e \in \delta(i)} x_e = 2, && i \neq 1, \\
 & && \sum_{e \in \delta(1)} x_e = 2m, \\
 & && \sum_{e \in \delta(S)} x_e \geq 2 \left\lceil \frac{\sum_{i \in S} b_i}{Q} \right\rceil, && S \subset \mathcal{N} \setminus \{1\}, S \neq \emptyset, \\
 & && x_e \in \{0, 1\}.
 \end{aligned}$$

Prob 2

Exercise 10.10

Let $\mathcal{T} = \{0, 1, \dots, T\}$, where T is an upper bound on the total time to complete all jobs. Each job consists of m phases denoted by (j, r) , $r = 1, \dots, m$. The r th phase of job j is processed by machine $M_{j(r)}$. Let $J = \{(j, r) \mid j = 1, \dots, n, r = 1, \dots, K\}$. The processing time of the r th phase of job j

is then $p_{j(r),j}$. Let $x_{j,r,t}$ be equal to 1, if the r th phase of job j is being processed at time t , and 0 otherwise. Let $C_{j,r}$ be the completion time of the r th phase of job j . Then, the objective is

$$\text{minimize} \quad \sum_j C_{j,m}.$$

Clearly, the total time phase r of job j spends in the system equals its processing time, i.e.,

$$\sum_{t \in \mathcal{T}} x_{j,r,t} = p_{j(r),j}.$$

In addition,

$$\begin{aligned}
 \sum_{t \in \mathcal{T}} t x_{j,r,t} &= \sum_{t=C_{j,r}-p_{j(r),j}}^{C_{j,r}} t \\
 &= C_{j,r}(1 + p_{j(r),j}) - \frac{p_{j(r),j}(1 + p_{j(r),j})}{2}.
 \end{aligned}$$

The precedence constraints are modeled as follows:

$$C_{j,r} \geq C_{j,r-1} + p_{j(r),j}.$$

Let $M_k = \{(j, r) \mid \text{the } r\text{th phase of job } j \text{ is processed by machine } k\}$. Because each machine should process at most one job at every time

$$\sum_{(j,r) \in M_k} x_{j,r,t} \leq 1, \quad \forall t \in \mathcal{T}.$$

3)

$$X_{ij} = \begin{cases} 1 & \text{if travel from city } i \text{ to } j \\ 0 & \text{o.w.} \end{cases}$$

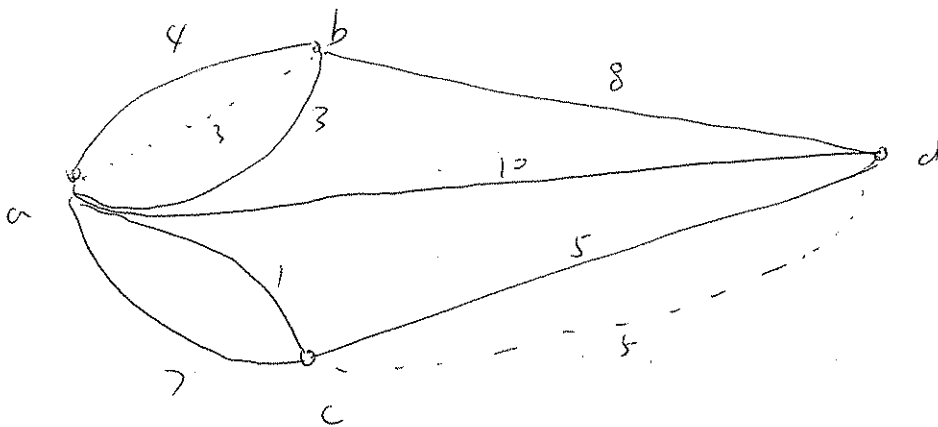
$$\text{min } \sum_{i=1}^5 \sum_{j=1}^5 C_{ij} X_{ij}$$

$$\text{s.t. } \sum_{\substack{j=1 \\ j \neq i}}^5 X_{ij} = 1 \quad i=1, 2, \dots, 5$$

$$\sum_{\substack{j=1 \\ j \neq i}}^5 X_{ji} = 1 \quad i=1, 2, \dots, 5$$

$$\sum_{\substack{i \in S \\ j \notin S}} X_{ij} \geq 1 \quad \text{for any } S \subseteq \{1, 2, 3, 4, 5\}, \quad 2 \leq |S| \leq 3$$

4) Every node has odd degree, so we could use coupling.



optimal path: a b a b d a c d c a

optimal cost: 46