

# HW 11

①

Phase I

$$\begin{array}{c|cccccc} 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ \hline 2 & 1 & 1 & -2 & 5 & 1 & 0 \\ 3 & 1 & 1 & -1 & 0 & 0 & 1 \end{array}$$

$$\begin{array}{c|cccccc} -5 & -2 & -2 & 3 & -5 & 0 & 0 \\ \hline 2 & \textcircled{1} & 1 & -2 & 5 & 1 & 0 \\ 3 & 1 & 1 & -1 & 0 & 0 & 1 \end{array}$$

$$\begin{array}{c|cccccc} -1 & 0 & 0 & -1 & 5 & 2 & 0 \\ \hline 2 & 1 & 1 & -2 & 5 & 1 & 0 \\ 1 & 0 & 0 & \textcircled{1} & -5 & -1 & 1 \end{array}$$

$$\begin{array}{c|cccccc} 0 & 1 & 2 & 3 & 4 & 1 & 1 \\ \hline 4 & 1 & 1 & 0 & -5 & 1 & 2 \\ 1 & 0 & 0 & 1 & -5 & -1 & 1 \end{array}$$

Phase II

$$\begin{array}{c|cccc} -7 & 0 & 1 & 0 & 24 \\ \hline 4 & 1 & 1 & 0 & -5 \\ 1 & 0 & 0 & 1 & -5 \end{array}$$

$$x^* = (4, 0, 1, 0)$$

$$\textcircled{2} \quad x_2: (2-2\lambda) - \begin{pmatrix} 1+2\lambda \\ 3+3\lambda \end{pmatrix}^T \begin{pmatrix} 1 \\ 0 \end{pmatrix} \geq 0 \Rightarrow \lambda \leq \frac{1}{4}$$

$$x_4: (4+\lambda) - \begin{pmatrix} 1+2\lambda \\ 3+3\lambda \end{pmatrix}^T \begin{pmatrix} -5 \\ 6 \end{pmatrix} \geq 0 \Rightarrow \lambda \leq -\frac{12}{13}$$

$$\text{So } -\frac{12}{13} \leq \lambda \leq \frac{1}{4}$$

$$\textcircled{3} \quad b = \begin{pmatrix} 2+\alpha \\ 3-\alpha \end{pmatrix} \quad B^{-1} = \begin{pmatrix} -1 & 2 \\ -1 & 1 \end{pmatrix}$$

$$B^{-1}b \geq 0 \Rightarrow \begin{pmatrix} \alpha-3\alpha \\ 1-2\alpha \end{pmatrix} \geq 0 \Rightarrow \alpha \leq \frac{1}{2}$$

Problem 4 (Exercise 10.2 in the book)

**Exercise 10.2**

Let  $x_i$  equal 1 if player  $i$  is selected, 0 otherwise. The problem can be formulated as follows.

$$\begin{aligned}
 &\text{maximize} && \sum_{i=1}^{20} s_i x_i \\
 &\text{subject to} && \sum_{i=1}^{20} x_i = 12 \\
 & && x_1 + \dots + x_5 \geq 3 \\
 & && x_4 + \dots + x_{11} \geq 4 \\
 & && x_9 + \dots + x_{16} \geq 4 \\
 & && x_{16} + \dots + x_{20} \geq 3 \\
 & && x_4 + x_8 + x_{15} + x_{20} \geq 2 \\
 & && \sum_{i=1}^{20} r_i x_i \geq 12r \\
 & && \sum_{i=1}^{20} a_i x_i \geq 12a \\
 & && \sum_{i=1}^{20} s_i x_i \geq 12s \\
 & && \sum_{i=1}^{20} h_i x_i \geq 12h \\
 & && \sum_{i=1}^{20} d_i x_i \geq 12d \\
 & && x_5 + x_{19} \leq 1 \\
 & && x_2 - x_{19} = 0 \\
 & && x_1 + x_7 + x_{12} + x_{16} \leq 3 \\
 & && x_i \in \{0, 1\}.
 \end{aligned}$$