550.661 Homework Due: November 10, 2009

Problem 1: Use the dual simplex method to solve the following linear program and its dual:

$$\begin{array}{lll} \min & 8x_1 + 2x_2 + 3x_3 & \\ \text{s.t.} & -4x_1 - 2x_2 & \leq -1 \\ & -2x_1 + 4x_2 + 4x_3 & \leq 3 \\ & 3x_1 - 4x_2 - 2x_3 & \leq 4 \\ & x_i \geq 0 \text{ for all i.} & \end{array}$$

Problem 2 Use the simplex method to solve the dual problem of the linear program in Problem 1. Notice that you should convert the dual to standard form first.

Problem 3: Check if the vector $(3, 1)$ is an optimal solution to the following linear program. If it is, find an optimal solution to its dual:

$$\begin{array}{lll} \min & -x_1 + x_2 & \\ \text{s.t.} & -2x_1 + x_2 & \leq 2 \\ & x_1 - 2x_2 & \leq 1 \\ & x_1 + x_2 & \leq 4 \\ & x_1, x_2 \geq 0. & \end{array}$$

Problem 4: Find all the values of α and β such that the following linear program has an optimal basic solution with basic variables x_3 and x_4:

$$\begin{array}{lll} \min & \alpha x_1 + 13x_2 - \alpha x_3 - 3x_4 & \\ \text{s.t.} & x_1 + x_2 - 2x_3 + 3x_4 & = -1 \\ & 2x_1 - x_2 - 3x_3 + 4x_4 & = \beta \\ & x_i \geq 0 \text{ for all i.} & \end{array}$$