

Homework 8 of 550.661, Due: November 3, 2009

Solve the following linear programs in Problems 1 and 2 by a two-phase simplex method:

**Problem 1:**

$$\begin{array}{ll} \min & -3x_1 - x_2 \\ \text{s.t.} & -x_1 - x_2 - 2x_3 + 3x_4 = -2 \\ & 2x_1 + x_2 + 3x_3 - 2x_4 = 3 \\ & -x_2 - x_3 + 4x_4 = -1 \\ & x_i \geq 0 \quad \text{for all } i. \end{array}$$

**Problem 2:**

$$\begin{array}{ll} \min & -3x_1 + x_2 \\ \text{s.t.} & 2x_1 + x_2 \geq 2 \\ & x_1 + 3x_2 \leq 3 \\ & x_2 \leq 4 \\ & x_1, x_2 \geq 0. \end{array}$$

**Problem 3:** Solve the linear program in Problem 3 by the big-M method.

**Problem 4:** Let  $(\hat{x}, \hat{y})$  be an optimal solution obtained after applying the big-M method. Show (a) if the artificial variables  $\hat{y} = 0$  then  $\hat{x}$  is an optimal solution to the original linear program; (b) if  $\hat{y} \neq 0$  then the original linear program is infeasible.