Homework 7 of 550.661, Due: October 27, 2009

Problem 1: Show that a departing basic variable can not be an entering variable in the very next iteration. (Notice that it could be an entering variable in some later iterations.)

Problem 2: Use the simplex method to solve the following problem, where you may use Bland's rule to resolve the degeneracy:

$$\begin{array}{lll} \min & -10x_1 + 57x_2 + 9x_3 + 24x_4 & \\ \text{s.t.} & 0.5x_1 - 5.5x_2 - 2.5x_3 + 9x_4 & \leq 0 \\ & 0.5x_1 - 1.5x_2 - 0.5x_3 + \ x_4 & \leq 0 \\ & x_1 & \leq 1 \\ & x_1,\ x_2,\ x_3,\ x_4 & \geq 0. \end{array}$$

Problem 3: Use the revised simplex method to solve the following problem

$$\begin{array}{lll} \min & 3x_1 - 4x_2 - x_3 - 2x_4 - 3x_5 & \\ \text{s.t.} & x_1 + x_2 + x_3 - x_4 + 2x_5 & \leq 12 \\ & x_1 - 2x_2 - x_3 - x_4 - x_5 & \geq -30 \\ & x_i \geq 0, \quad (i = 1, \cdots, 5). & \end{array}$$

Problem 4: Consider the following linear program:

$$\begin{array}{lll} \min & -4x_1 - x_2 + x_3 & \\ & x_1 - x_2 - 2x_3 & \leq 0 \\ & 2x_1 + 2x_2 & \leq 1 \ . \\ & 3x_1 + x_2 + x_3 & \leq 0 \\ & x_1,\ x_2,\ x_3 & \geq 0 \end{array}$$

At the k-th iteration, we have basic variables $\{x_2, x_5, x_3\}$ and

$$B^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ -2 & 1 & -4 \\ -1 & 0 & -1 \end{bmatrix}.$$

Carry out one step of the revised simplex method to find the new B^{-1} and basic variables.