

Homework 6, Due: October 20, 2009

Problem 1 Let a basic direction (generator) d at a vertex \hat{x} be given as follows:

$$d = \begin{bmatrix} e_i \\ -B^{-1}a_i \end{bmatrix}.$$

Suppose that the reduced cost $c^T d$ at \hat{x} with respect to this basic direction is negative and $B^{-1}a_i \leq 0$. Show that the linear program has no optimal solution because of unboundedness.

Problem 2 Use an example to show that the feasible-direction cone and the cone generated by the basic directions at a **degenerate** vertex may not be the same. Can you show that if all reduced costs ≥ 0 at a vertex then the vertex, degenerate or not, is still optimal?

Problem 3 Implement the simplex method to solve the following linear program without tableaux; that is, you compute basic directions, reduced costs and search direction and stepsize iteratively. Use a graph to show the progress of the computation:

$$\begin{array}{ll} \min & -120x_1 - 100x_2 \\ \text{s.t.} & x_1 + 3x_2 \leq 9 \\ & 2x_1 + 3x_2 \leq 12 \\ & x_1 \geq 0, \quad x_2 \geq 0. \end{array}$$

Problem 4 Implement the simplex method with tableaux to solve the following problem:

$$\begin{array}{ll} \min & -2x_1 - 4x_2 - x_3 - x_4 \\ \text{s.t.} & x_1 + 3x_2 + x_4 \leq 4 \\ & 2x_1 + x_2 \leq 3 \\ & x_2 + 4x_3 + x_4 \leq 3 \\ & x_i \geq 0, \text{ for all } i \end{array}$$