**Homework 4 (revised version), Due: October 6, 2009**

**Problem 1:** Use tableaux to find all vertices of the feasible region and then use the enumeration method to solve the following linear program

$$\begin{array}{ll} \max & x_1 + 2x_2 \\ \text{s.t.} & x_1 + 3x_2 \ \leq 12 \\ & x_1 - x_2 \ \leq 3 \\ & x_1 \geq 0, \quad x_2 \geq 0. \end{array}$$

**Problem 2:** (Problem 3.4 in Textbook) Consider the polyhedron $P = \{x \in R^n \ : \ Ax = b,\ Dx \leq f, Ex \leq g\}$. Let $\hat{x}$ be an element of $P$ that satisfies $D\hat{x} = f$ and $E\hat{x} < g$. Show that the set of feasible directions at $\hat{x}$ is the set

$$\{\ d \ : Ad = 0,\ Dd \leq 0\}.$$

**Problem 3:** Let $P = \{x \in R^3 \ : x_1 + x_2 + x_3 = 1, x_i \geq 0 \text{ for all } i\}$. Find the feasible direction cone of the set at $x = (0, 0, 1)$ and also find its generators.

**Problem 4:** Consider the following linear program:

$$\begin{array}{ll} \min & -x_1 - 2x_2 \\ \text{s.t.} & x_1 + x_2 \ \leq 6 \\ & 2x_1 - x_2 \ \leq 6 \\ & x_1 \geq 0,\ x_2 \geq 0. \end{array}$$

(1) Use graph to find all the vertices and its feasible-direction cones in terms of their generators.
(2) check for each vertex if it is an optimal solution by showing that the objective function will not decrease for any feasible direction at this vertex.