

Homework 3, Due: September 29, 2009

Problem 1: Find all the vertices of the following polyhedron and for each vertex, list all its adjacent vertices:

$$S = \{x \in \mathbb{R}^3 : \begin{array}{rcl} 3x_1 + x_2 + x_3 & \leq & 5 \\ x_1 + x_2 - x_3 & = & 1 \\ x_2 & \geq & -3 \\ x_1 \geq 0, x_3 \geq 0 & & \end{array} \}$$

Problem 2: Transform the polyhedron in Problem 1 to standard form and use tableaux to compute all the vertices.

Problem 3: Let P be the convex hull of the following vectors:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \end{bmatrix}.$$

Find the matrix A and the vector b such that $P = \{x : Ax \leq b\}$. What are the vertices of the polyhedron?

Problem 4: Consider a polyhedron $P = \{x : Ax \leq b\}$ in \mathbb{R}^n . Set up a linear program for finding the smallest rectangle with sides parallel to the coordinate axes that contains the polyhedron.