Homework Due: December 1, 2009

Problem 1: **(The vehicle routing problem)** An undirected graph $G = (\mathcal{N}, \mathcal{E})$ represents a transportation network. Node $i \in \mathcal{N}$, for $i \neq 1$, represents customers with demand of b_i units. The travel costs are d_e for every arc $e \in \mathcal{E}$. A company has m vehicles, each of capacity Q, that need to visit all customers in order to satisfy demand. Each vehicle is to follow a route that starts at a central depot (node 1), visits some customers, and returns to the depot. Suppose that the demand of each customer can be carried by a single vehicle, i.e., $b_i \leq Q$ for all i. Assuming that the demand of any customer cannot be divided into several vehicles, formulate the problem of constructing routes for the vehicles that minimize the total transportation cost.

Problem 2: **(Job shop scheduling)** A factory consists of m machines $M_1, \ldots, M_m$, and needs to process n jobs every day. Job j needs to be processed once by each machine in the order $(M_{j(1)}, \ldots, M_{j(m)})$. Machine M_i takes time p_{ij} to process job j. A machine can only process one job at a time, and once a job is started on any machine, it must be processed to completion. The objective is to minimize the sum of the completion times of all the jobs. Provide an integer programming formulation for this problem.

Problem 3: Set up an integer program for solving the Traveling Salesman Problem with 5 cities.

Problem 4: Solve the Chinese Postman Problem with the following graph: