Homework 11 Due: November 24, 2009

Problem 1: Use the simplex method to solve the following linear program:

$$\begin{array}{lll} \min & x_1 + 2x_2 + 3x_3 + 4x_4 & \\ \text{s.t.} & x_1 + x_2 - 2x_3 + 5x_4 & = 2 \\ & x_1 + x_2 - x_3 & = 3 \\ & x_1,\, x_2,\, x_3,\, x_4 & \geq 0. \end{array}$$

Problem 2 Find all the values of λ such that the optimal solution of Problem 1 remains optimal if the objective function is replaced by $(1+2\lambda)x_1+(2-2\lambda)x_2+(3+3\lambda)x_3+(4+\lambda)x_4$.

Problem 3 Find all the values of α such that the optimal basis matrix B in Problem 1 remains optimal after the right-hand-side is replaced by $(2+\alpha, 3-\alpha)$.

Problem 4: **(Selection of the dream team)** The coach of the national basketball team is faced with the decision of selecting 12 players for the upcoming international tournament. He has limited his final selection to 20 players, $p_1, \ldots, p_{20}$. For each player, the coach has collected several statistics that can be summarized as follows. His rebounding average r_i, his assists average a_i, his height h_i, his scoring average s_i, and his overall defense ability d_i. The players have been divided into four broad categories: play makers (PM) $(p_1, \ldots, p_5)$, shooting guards (SG) $(p_4, \ldots, p_{11})$, forwards (F) $(p_9, \ldots, p_{16})$, and centers (C) $(p_{16}, \ldots, p_{20})$. Notice that there are players that can be used in multiple roles (for example player p_4 can be used both as a play maker and a shooting guard). Players p_4, p_8, p_{15}, p_{20} play in the NCAA (college level), while all of the rest play in the NBA (professional level). For balance purposes, the team should consist of at least 3 play makers, 4 shooting guards, 4 forwards, and 3 centers, which implies that some players with dual roles should be selected. In addition, at least 2 players from the NCAA should be selected, while the mean rebounding, assists, scoring average, height, and defense ability should be at least r, a, s, h, d, respectively. The problem is further complicated by the fact that there are compatibility problems among some of the players. Player p_5 has declared that if player p_9 is selected, then he does not want to be in the team. Also, players p_2 and p_{19} can only be selected together as they play in the same team for years and feel that they are much more effective together. Finally, at most 3 players from the same team should be selected, so that the coach is not accused of favoritism (players p_1, p_7, p_{12}, p_{16} play for the same team). Faced with these difficulties, the coach has decided that he would like to maximize the scoring average, while satisfying the various constraints. Formulate the problem that the coach is facing as an integer programming problem.