Homework 10 Due: November 17, 2009

Problem 1: Use duality to check if vector x=(1,0,1,0) is an optimal solution of the following linear program:

$$\begin{array}{lll} \min & -x_1 + 2x_2 - x_3 - x_4 & \\ \text{s.t.} & x_1 + x_2 - x_3 + 2x_4 & \geq -2 \\ & x_1 + 2x_2 - x_3 + x_4 & = 0 \\ & -x_1 - x_2 - x_3 - x_4 & \geq -2 \\ & x_1, x_2, x_3 \geq 0, \quad x_4 & \text{unrestricted.} \end{array}$$

Problem 2: It is known that an optimal solution to the following linear program is of the form $(0, \alpha, \beta)$ with $\alpha > 0$ and $\beta > 0$. Without using any simplex pivots, solve the problem and its dual:

$$\begin{array}{lll} \min & -12x_1 + 10x_2 + 2x_3 & \\ & -4x_1 + x_2 - 8x_3 & \geq 1 \\ & -x_1 + x_2 + 12x_3 & \geq 3 \\ & x_1, x_2, x_3 \geq 0. & \end{array}$$

Problem 3: Use Farkas' lemma to prove the following theorem of alternative: If A is an $m \times n$ matrix and $b \in R^m$, exactly one of the following two systems has a solution:

$$(1) \quad Ax = b, \quad (2) \quad A^T w = 0, \quad b^T w = 1.$$

Problem 4: Find all the values of α and β such that the following linear program has an optimal solution with basic variables x_1 and x_4:

$$\begin{array}{lll} \min & x_1 + \beta x_2 - x_3 - x_4 & \\ \text{s.t.} & x_1 + \quad \beta x_3 + x_4 & = 4 + \alpha \\ & \quad x_2 + x_3 + x_4 & = 2 + 2\alpha \\ & x_i \geq 0 & \forall i. \end{array}$$