Final Exam of 550.661

Hand in the exam personally to Room 201 Whitehead Hall between 10:00 A.M. - 12:00 Noon., December 10, 2009.

The solutions I provided in this exam were completely done by myself, without any help from any other people.

(name) (signature)

1. Use the revised simplex method to solve the following problem:

$$\begin{array}{lll} \min & x_1 - 8x_2 & \\ \text{s.t.} & -x_1 + x_2 & \leq 1 \\ & 2x_1 - x_2 & \leq 0 \\ & x_1 + x_2 & \leq 5 \\ & x_l, \quad x_2 & \geq 0. \end{array}$$

2. It is known that $x = (3, 1)$ and $w = (0, \frac{2}{3}, \frac{1}{3})$ are optimal solutions to the following linear program and its dual, respectively. Find the values of α, β and γ. Justify your answer.

$$\begin{array}{lll} \max & \alpha x_1 - x_2 & \\ \text{s.t.} & -2x_1 + x_2 & \leq 2 \\ & x_1 + \beta x_2 & \leq 1 \\ & \gamma x_1 + x_2 & \leq 4 \\ & x_1, \quad x_2 & \geq 0. \end{array}$$

3. Use the simplex method to solve the following linear program and find the all the values of λ such that the optimal solution remains optimal if the objective function is replaced by $(1 + 2\lambda)x_1 + (2 - 2\lambda)x_2 + (3 + 3\lambda)x_3 + (4 + \lambda)x_4$:

$$\begin{array}{lll} \min & x_1 + 2x_2 + 3x_3 + 4x_4 & \\ \text{s.t.} & x_1 + x_2 - 2x_3 + 5x_4 & = 2 \\ & x_1 + x_2 - x_3 & = 3 \\ & x_1, x_2, x_3, x_4 & \geq 0. \end{array}$$

4. Use the Gomory cutting plane method to solve the following integer program:

$$\begin{array}{ll} \min & x_1 + x_2 + x_3 + x_4 \\ \text{s.t.} & 4x_1 - \frac{34}{5}x_2 + \frac{1}{2}x_3 + 2x_4 + x_5 \quad = \frac{5}{2} \\ & x_1 + \frac{6}{5}x_2 + \frac{1}{2}x_3 + 3x_4 \quad + x_6 \quad = \frac{3}{2} \\ & x_1, \cdots x_6 \geq 0 \quad \text{integers.} \end{array}$$

5. Use a two-phase method or the big-M method to solve the following problem:

$$\begin{array}{lll} \min & -x_1 + 2x_2 + 3x_3 + x_4 + x_5 - 2x_6 & \\ \text{s.t.} & x_1 + 2x_2 + 2x_3 + x_4 + x_5 & = 12 \\ & x_1 + 2x_2 + x_3 + x_4 + 2x_5 + x_6 & = 18 \\ & 3x_1 + 6x_2 + 2x_3 + x_4 + 3x_5 & = 24 \\ & x_i \geq 0 \text{ for all i.} & \end{array}$$

6. Use the duality to check if the vector $(5/26, 5/2, 27/26)$ is an optimal solution to the following linear program.

$$\begin{array}{lll} \max & 9x_1 + 14x_2 + 7x_3 & \\ \text{s.t.} & 2x_1 + x_2 + 3x_3 & \leq 6 \\ & 5x_1 + 4x_2 + x_3 & \leq 12 \\ & \quad 2x_2 & \leq 5 \\ & x_1, x_2, x_3 \quad \text{unrestricted.} & \end{array}$$

7. Use Farkas' lemma to prove the following theorem of alternatives: Exactly one of the following systems has solutions:

$$(1) \left\{ \begin{array}{l} Ax \leq b \\ x \geq 0. \end{array} \right. \qquad (2) \left\{ \begin{array}{l} A^T y \geq 0 \\ b^T y < 0 \\ y \geq 0. \end{array} \right.$$

(Hint: Insert slack variables.)