You may discuss ideas for the homework with other students in the class, but when it comes time to write up your solutions you must do it alone, and your write-up must exactly reflect your understanding. You may not consult other textbooks, online resources, nor any existing solutions.

**Problem 1:** Consider the following nonlinear programming problem:
\[ \begin{align*}
\min & \quad \frac{x_1^2}{2} + \frac{x_2^2}{2} \\
\text{s.t.} & \quad x_1 + x_2 + 1 \leq 0.
\end{align*} \]
Find all KKT points; also write the associated KKT multipliers and objective function values.

**Problem 2:** Suppose \( A \in \mathbb{R}^{n \times n} \) is symmetric positive definite, \( b \in \mathbb{R}^n \) is nonzero, \( c \in \mathbb{R} \) is positive, and the variables will be \( x \in \mathbb{R}^n \). Consider the following nonlinear programming problem:
\[ \begin{align*}
\min & \quad \frac{1}{2} x^T A x \\
\text{s.t.} & \quad b^T x + c \leq 0.
\end{align*} \]
Find all KKT points; also write the associated KKT multipliers and objective function values. (Note: Problem 1 is a special case of this problem.)

**Problem 3:** Consider the following nonlinear programming problem (P):
\[ \begin{align*}
\min & \quad \frac{x_1^2}{2} + \frac{x_2^2}{2} \\
\text{s.t.} & \quad x_1 + x_2 + 1 \leq 0.
\end{align*} \]

a) Write the Lagrangian dual (DP) \( \sup_{\lambda \geq 0} \theta(\lambda) \). Your task is to simplify \( \theta \) as much as possible.

b) Solve (DP) by directly maximizing \( \theta \).

c) Is there a duality gap?

**Problem 4:** Suppose \( A \in \mathbb{R}^{n \times n} \) is symmetric positive definite, \( b \in \mathbb{R}^n \) is nonzero, \( c \in \mathbb{R} \) is positive, and the variables \( x \in \mathbb{R}^n \). Consider the following nonlinear programming problem (P):
\[ \begin{align*}
\min & \quad \frac{1}{2} x^T A x \\
\text{s.t.} & \quad b^T x + c \leq 0.
\end{align*} \]

a) Write the Lagrangian dual (DP) \( \sup_{\lambda \geq 0} \theta(\lambda) \). Your task is to simplify \( \theta \) as much as possible.

b) Solve (DP) by directly maximizing \( \theta \).

c) Is there a duality gap?

**Problem 5:** Consider the linear program (P) \( \min c^T x \) s.t. \( Ax \geq b, \ x \geq \vec{0} \), where \( A \in \mathbb{R}^{m \times n} \), \( b \in \mathbb{R}^m \), and \( c \in \mathbb{R}^n \). Compute and simplify the Lagrangian Dual (DP) using \( S = \mathbb{R}^n \). (In other words, all of the constraints should be part of \( \vec{g} \).)
Problem 6: Consider the program (P) \( \min f(x) \) s.t. \( \vec{g}(x) \leq \vec{0}, \vec{h}(x) = \vec{0}, x \in S \), where \( S \subseteq \mathbb{R}^n \) is an open set, and \( f \) and each component of \( \vec{g} \) and \( \vec{h} \) is continuously differentiable \( S \rightarrow \mathbb{R} \). Write the KKT conditions for (P) specifically by converting this mixed-equality-inequality form of (P) to the inequality-form and simplifying the KKT conditions from the inequality-form.