
1. Shows that eigenvalues \( \lambda \) for the following special matrix types satisfy the stated properties

- (i) \( \mathbf{A} \) unitary \( \Rightarrow |\lambda| = 1 \)
- (ii) \( \mathbf{A} \) Hermitian \( \Rightarrow \lambda \) real
- (iii) \( \mathbf{A} \) orthogonal, \( n \) odd \( \Rightarrow \) at least one \( \lambda = \pm 1 \)
- (iv) \( \mathbf{A} \) projection \( \Rightarrow \lambda = 0, 1 \)

2. Consider the vector \( \mathbf{x} = [1, 2, 3, 4]^\top \). Calculate directly from the definitions the norms \( \|\mathbf{x}\|_1, \|\mathbf{x}\|_2, \|\mathbf{x}\|_\pi, \|\mathbf{x}\|_\infty \) for this vector \( \mathbf{x} \). Check your results with the function `norm(x,p)` in Matlab.

3. Show that \( \lim_{p \to \infty} \|\mathbf{x}\|_p = \|\mathbf{x}\|_\infty \).

   \textbf{Hint:} Write \( \sum_{i=1}^n |x_i|^p = |x_m|^p \sum_{i=1}^n |x_i/x_m|^p \), where \( m \) is the smallest index \( i \) such that \( |x_m| = \max_i |x_i| \).

4. Consider the following two matrices

- (i) \( \mathbf{A} = \begin{pmatrix} 1 & 4 \\ 8 & 5 \end{pmatrix} \)
- (ii) \( \mathbf{A} = \begin{pmatrix} 13 & -8 \\ 20 & 5 \end{pmatrix} \)

   For each of these choices of the matrix \( \mathbf{A} \), calculate directly from the definitions the quantities \( \rho(\mathbf{A}), \|\mathbf{A}\|_F, \|\mathbf{A}\|_1, \|\mathbf{A}\|_2, \|\mathbf{A}\|_\infty \). Check your results with the functions `eig` and `norm(A,p)` in Matlab.

5. Prove that the Frobenius norm is a proper matrix norm and that it is consistent with the \( \ell_2 \) vector norm \( \| \cdot \|_2 \).

6. Prove that \( \|\mathbf{A}\|_1 = \max_{1 \leq j \leq n} \sum_{i=1}^n |a_{ij}| \).

   \textbf{Hint:} Use an argument similar to that employed for Theorem 7.11 in the text.

7. Consider the following two matrices

- (i) \( \mathbf{A} = \begin{pmatrix} 7/16 & -3/16 \\ -1/16 & 5/16 \end{pmatrix} \)
- (ii) \( \mathbf{A} = \begin{pmatrix} 7/2 & 3/2 \\ -3 & -1 \end{pmatrix} \)

   (a) For each of these choices, calculate directly from the definitions the spectral radius \( \rho(\mathbf{A}) \) and the inverse matrix \( \mathbf{B}^{-1} \) for \( \mathbf{B} = \mathbf{I} - \mathbf{A} \).

   (b) Write a code in Matlab or other language to evaluate the matrix \( \mathbf{B}_N^{-1} \) defined by

   \[ \mathbf{B}_N^{-1} \equiv \sum_{k=0}^N \mathbf{A}^k \]

   and find the least \( N \leq 32 \) (if any) for which \( \|\mathbf{B}^{-1} - \mathbf{B}_N^{-1}\|_2 < 10^{-8} \). Explain your numerical observations for both matrices using the results in part (a).