1. Follow the instructions of Strogatz, Problems 4.1.2-7 for the system
   \[ \dot{\theta} = \cos(\theta) \sin(3\theta). \]

2. [Double Points] (a) Consider the vector field on the circle given by \( \dot{\theta} = \sin(\theta). \) Show that this system has a single-valued potential \( V(\theta), \) i.e. for each point on the circle, there is a well-defined value of \( V \) such that \( \dot{\theta} = -dV/d\theta. \) (As usual, \( \theta \) and \( \theta + 2\pi k \) are to be regarded as the same point on the circle, for each integer \( k. \))
   (b) Now consider \( \dot{\theta} = 1. \) Show that there is no single-valued potential \( V(\theta) \) for this vector field on the circle.
   c) What’s the general rule? When does \( \dot{\theta} = f(\theta) \) have a single-valued potential?
   d) In Homework #2, Problem 4, you were asked to give an analytical proof that periodic solutions are impossible for vector fields on the line. Explain why your arguments there do not carry over to vector fields on the circle. Specifically, what part of the argument fails?

3. Follow the instructions of Strogatz, Problem 4.2.2 for the function
   \[ x(t) = \cos(10t) + \cos(11t). \]

4. Strogatz, Problem 4.3.2. See the book’s sketch of the solution, p.457, for some basic steps.

5. Follow the instructions of Strogatz, Problems 4.3.3-8 for the system
   \[ \dot{\theta} = \mu \sin \theta - \sin^3 \theta. \]

6. [Double Points] Strogatz, Problem 4.5.2. In addition to the general case, consider the particular set of functions for any \( \alpha \geq 1 \) defined by
   \[ f_\alpha(\theta) = \frac{\pi}{2} \times \begin{cases} 1 - \left| 1 - \frac{2\theta}{\pi} \right|^\alpha, & 0 < \theta \leq \pi \\ 1 + \left| 1 + \frac{2\theta}{\pi} \right|^\alpha - 1, & -\pi < \theta \leq 0 \end{cases}, \]
   which for \( \alpha = 1 \) coincides with the triangle wave function in Strogatz, Problem 4.5.1. In part (a), graph \( f_\alpha(\theta) \) for \( \alpha = 3/2, 2, 3. \) In part (e), use the arguments of Strogatz, Problems 4.3.9-10 to find a formula for \( T_{drift}. \)