Midterm 550.391, Oct. 11, 2007.

Do all of the following **three** *problems.* Show all your work. Answers without supporting work may receive no credit.

I attest that I have completed this exam without unauthorized assistance from any person, materials, or device:

Full name: _____

Signature: _____

(See the Johns Hopkins Handbook *Academic Ethics for Undergraduates*).

1. Consider the vector field

$$\dot{x} = \ln |x|$$

on the whole real line $-\infty < x < +\infty$.

(a) Find all the fixed points of the system.

(b) Calculate the linearization around every fixed point and determine its stability.

(c) Calculate the potential $V(x)$ for the system and sketch its graph.

(d) Sketch the phase portrait of the system on the real line.

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2. Consider the equation

$$\dot{x} = rx - x\ln(1+x)$$

with parameter r, on the interval $-1 < x < +\infty$.

(a) For each real value of r, find all the fixed points of this system. You should be able to find explicit expressions for these.

(b) Find the critical value $r = r_*$ at which a bifurcation occurs and determine the stability of all of the fixed points both for $r < r_*$ and $r > r_*$.

(c) Identify the type of bifurcation and sketch the bifurcation diagram.

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3. Consider the system

$$\dot{x} = rx + x^3,$$

where $r > 0$ is fixed.

(a) Show that $x(t) \to \pm\infty$ in finite time, starting from any initial condition $x_0 \neq 0$. *Hint:* You should be able to find a simple "comparison system" for which this is true.

(b) Suppose that a stabilizing nonlinearity is added. To be specific, consider the altered system

$$\dot{x} = rx + x^3 - x^5.$$

Is it still true that $x(t) \to \pm\infty$? Justify your answer.

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