Midterm 550.391, Oct. 12, 2009.

Do all of the following **three** *problems.* Show all your work. Answers without supporting work may receive no credit.

I attest that I have completed this exam without unauthorized assistance from any person, materials, or device:

Full name: _____

Signature: _____

(See the Johns Hopkins Handbook *Academic Ethics for Undergraduates*).

1. Consider the vector field

$$\dot{x} = 1 - 2x + x^2$$

on the whole real line $-\infty < x < +\infty$.

(a) Find all the fixed points of the system.

(b) Determine the stability of each fixed point. State whether linearization is useful or not for this purpose.

(c) Calculate the potential $V(x)$ for the system and sketch its graph.

(d) Sketch the phase portrait of the system on the real line.

.

2. Consider the equation

$$\dot{x} = rx - \cosh x$$

with parameter r, on the interval $-\infty < x < +\infty$.

(a) Show graphically that bifurcations occur in this system at two critical values (r_c, x_c). Determine the stability of any fixed points that occur for all values of r.

(b) Derive an analytical equation for the values x_c (but don't solve the equation).

(c) Identify the type of bifurcations and qualitatively sketch the bifurcation diagram in the (r, x) plane.

(d) Derive a formula $r(x)$ that quantitatively gives the bifurcation diagram.

.

3. Consider the system

$$\dot{x} = (\cos t)x.$$

(a) Verify that the periodic function $x(t) = x_0 \exp(\sin t)$ is the exact solution for initial condition x_0.

(b) Show that the system has a time-dependent potential $V(x,t)$ such that

$$\dot{x} = -\frac{\partial V}{\partial x}(x,t)$$

and find the explicit formula for $V(x,t)$.

(c) Calculate the time-derivative $(d/dt)V(x(t),t)$. Use this result to explain why there is no contradiction between (a) and (b).

.