Midterm 550.391, Oct. 4, 2010.

Do all of the following **three** *problems.* Show all your work. Answers without supporting work may receive no credit.

I attest that I have completed this exam without unauthorized assistance from any person, materials, or device:

Full name: _____

Signature: _____

(See the Johns Hopkins Handbook *Academic Ethics for Undergraduates*).

1. Consider the vector field

$$\dot{x} = 1 + \cos x$$

on the whole real line $-\infty < x < +\infty$.

(a) Find all of the fixed points of the system.

(b) Calculate the linearized dynamics around each fixed point. What does linearization imply for their stability?

(c) Using the vector field $f(x) = 1 + \cos x$, sketch the phase portrait of the system. What does the portrait imply for the stability of the fixed points?

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2. Consider the equation

$$\dot{x} = x^{1/5}$$

on the interval $-\infty < x < +\infty$ with a single equilibrium point at $x_* = 0$. Note that $x^{1/5}$ is the real 5th root, which satisfies $(-x)^{1/5} = -x^{1/5}$.

(a) Does the linearization exist at $x_* = 0$? If so, calculate it. Does the potential $V(x)$ exist? If so, calculate and sketch it.

(b) Use the results in (a) to determine the stability of $x_* = 0$ and to sketch the phase portrait of the system.

(c) Calculate the time $T(\epsilon)$ for a solution to start at $x = \epsilon > 0$ and end at $x = 1$ and then find $T(0) = \lim_{\epsilon \to 0} T(\epsilon)$. But the equilibrium point $x_* = 0$ stays fixed for all times, so shouldn't $T(0) = +\infty$? Explain this apparent contradiction.

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3. Consider the equation

$$\dot{x} = r^2 - x^2$$

with parameter r, on the interval $-\infty < x < +\infty$.

(a) Find all of the fixed points of this system for each real value of r and find the critical value $r = r_c$ at which a bifurcation occurs.

(b) Sketch the phase portraits for $r < r_c$, $r = r_c$, and $r > r_c$ and determine the stability of all of the fixed points in each case.

(c) Identify the type of bifurcation and sketch the bifurcation diagram.

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