Homework No.9, 550.391, Due November 29, 2011.

1. Strogatz, Problem 9.2.1.

2. Strogatz, Problem 9.2.3.

3. Strogatz, Problem 9.3.1.

4. Strogatz, Problem 9.3.6. Do not make a plot of x vs. z, as Strogatz requests, but instead make 3D plots of (x, y, z). You may use the `MATLAB` plotting option `odephas3` with integrators like `ode45` or else use `plot3` to plot output data arrays. Also, plot the time series of all three variables $x(t), y(t)$ and $z(t)$.

5. Strogatz, Problem 9.3.9. You can use one of the standard integrators in `MATLAB` such as `ode45` to obtain a solution matrix [`t1`, `y1`] for the Lorenz model. Make sure that your initial vector is on the attractor! Repeat with another nearby initial vector to create a second solution matrix [`t2`, `y2`]. Since the `MATLAB` integrators like `ode45` do not use a uniform time-step, it requires a little bit of extra work to compare two nearby trajectories. Even if one integrates over the same interval of time [`t0 tf`], the solution output vectors for two nearby initial conditions need not have the same length! To make the comparison, it is necessary to interpolate the outputs on a uniform grid of times `tt = t0 : dt : tf`, for example, by using spline interpolation in `MATLAB` with the command

```
yy1 = interp1(t1, y1, tt, 'spline');  yy2 = interp1(t2, y2, tt, 'spline');
```

Then type

```
dyy = yy2 - yy1;
```

to create the difference. The command

```
nn = sum(abs(dyy)');
```

will create a row vector of norms of the differences for all times. Then

```
plot(tt, log(nn));
```

will plot the logarithm of the norm versus time, and

```
polyfit(tt, log(nn), 1);
```

will give the slope and intercept of the best linear fit.

6. Maps of the unit interval $[0, 1]$ into itself defined by

$$x_{n+1} = \begin{cases} ax_n + b & 0 \le x_n \le 1/2 \\ a(1 - x_n) + b & 1/2 \le x_n \le 1 \end{cases}$$

for $1 < a \le 2,\ 0 \le b \le (2 - a)/2$ are known as "tent maps". They can be considered as simple analytical models of the Lorenz map discussed in Strogatz' Section 9.4.

(a) Show that the above formula indeed defines a map of the unit interval into itself. Why are these called "tent maps"?

(b) Show that there is a single fixed point x_* for these maps, with $x_* > 1/2$, and analyze its stability.

(c) Show that the tent maps have a period-2 orbit. Is it stable or unstable?

(d) Calculate analytically the exact value of the Lyapunov exponent λ of the tent map for each point in the interval $[0, 1]$.

7. Strogatz, Problem 9.5.4. Use instead $r = 24.4 + A \sin \omega t$ with $A = 3$ and $\omega = 2\pi/500$ and you will see the phenomenon more clearly.