Homework No.6, 550.391, Due October 28, 2011.

1.Find the fixed points for the system

$$\dot{x} = x - x^3, \quad \dot{y} = -y.$$

Sketch the nullclines, the vector field, and a plausible phase portrait.

2. Repeat Problem 1 for the system

$$\dot{x} = x^2 - y^3, \quad \dot{y} = x.$$

3. Strogatz, 6.1.11.

4. Strogatz, 6.2.1.

5. Find and classify the fixed points of the system

$$\dot{x} = xy^2 - 1, \quad \dot{y} = x^2 - y.$$

Sketch the neighboring trajectories and try to fill in the rest of the phase portrait.

6. For the system in Problem 5, plot a computer-generated phase portrait and compare to your approximate sketch.

7. Strogatz, 6.3.10.

8. Consider the following "rabbits vs. sheep" problem

$$\dot{x} = x(3 - x - y), \quad \dot{y} = y(2 - 2x - y),$$

where $x, y \geq 0$. Explain why this Lotka-Volterra system, in comparison with that of Section 6.4, can be considered a model of "aggressive rabbits" and "timid sheep". Find the fixed points, investigate their stability, draw the nullclines, and sketch plausible phase portraits. Indicate the basins of attraction of any stable fixed points. Explain, in ecological terms, why the outcome of this Lotka-Volterra system is different from that in Section 6.4.

9. Strogatz, 6.5.4.

10. Strogatz, 6.5.12.

11. Strogatz, 6.5.14. In part (b), make as complete an investigation as you can!

12. Strogatz, 6.8.1, parts (b),(e) and Strogatz, 6.8.4.

13. Strogatz, Problem 6.8.8. Show that at least one of the fixed points in the region bounded by C_1, C_2, and C_3 must, in fact, be a saddle.

14. Strogatz, Problem 6.8.12.