

Homework No.4, 550.391, Due October 14, 2011.

1. Find and classify all of the fixed points of the system

$$\dot{\theta} = \sin(\theta) - \cos(\theta).$$

Sketch the phase portrait on the circle.

2. [Double Points] (a) Consider the vector field on the circle given by $\dot{\theta} = \sin(\theta)$. Show that this system has a single-valued potential $V(\theta)$, i.e. for each point on the circle, there is a well-defined value of V such that $\dot{\theta} = -dV/d\theta$. (As usual, θ and $\theta + 2\pi k$ are to be regarded as the same point on the circle, for each integer k .)

(b) Now consider $\dot{\theta} = 1$. Show that there is no single-valued potential $V(\theta)$ for this vector field on the circle.

c) What's the general rule? When does $\dot{\theta} = f(\theta)$ have a single-valued potential?

d) In Homework # 2, Problem 4, you were asked to give an analytical proof that periodic solutions are impossible for vector fields on the line. Explain why your arguments there *do not* carry over to vector fields on the circle. Specifically, what part of the argument fails?

3. Graph $x(t) = \cos(10t) + \cos(11t)$ for $-25 < t < 25$. You should find that the amplitude of the oscillations is *modulated*—it grows and decays periodically.

a) What is the period of the amplitude modulations?

b) Solve this problem analytically, using a trigonometric identity that converts sums of sines and cosines to products of sines and cosines.

4. Strogatz, Problem 4.3.2. See the book's sketch of the solution, p.457, for some basic steps.

5. Draw the phase portrait of the system

$$\dot{\theta} = \mu \sin(\theta) - \sin(3\theta)$$

as a function of the control parameter μ . Classify the bifurcations that occur as μ varies, and find all the bifurcation values of μ .

6. (a-d) Strogatz, Problem 4.5.1.

(e) In part (d), why does the square root scaling law not apply near the bifurcation?