

Homework No.3, 550.391, Due September 30, 2011.

1. Show that a saddle-node bifurcation occurs for the system

$$\dot{x} = rx^2 - e^x$$

at a critical value r_c to be determined. Sketch all of the qualitatively different vector fields that occur as r varies and the bifurcation diagram of fixed points x_* versus r .

2. Repeat the previous problem, but show instead that a transcritical bifurcation occurs for the system

$$\dot{x} = x(\tanh x - r).$$

3. Repeat the previous problem, but show instead that a pitchfork bifurcation occurs for the system

$$\dot{x} = r \tanh x - \sinh x$$

Is the pitchfork supercritical or subcritical?

4. Strogatz, Problem 3.2.6
5. Strogatz, Problem 3.4.6.
6. Strogatz, Problem 3.4.12.
7. Strogatz, Problem 3.5.4.
8. Strogatz, Problem 3.6.2.
9. Strogatz, Problem 3.7.2.