

Ensembles & Probability

(a) Rationale for a Probabilistic Approach

We have already encountered probability theory a number of times in our discussion of spatial-coarse-graining and the closure problem. For example,

* In formal closure, new random noise terms are generated due to unknown initial conditions for eliminated (subscale) modes

* For molecular systems with scale-separation, the coarse-grained fields are deterministic and closed for scales $\max\{\epsilon, \lambda\} \ll \ell \ll L \nabla$, due to the law of large numbers.

In many cases, probabilistic information is required for practical applications. For example,

* The problem of structural failure of materials (discussed by L. Graham-Brady & K. Ramech in 560,700) needs probabilistic information to predict failure events

spatially-varying microscale properties

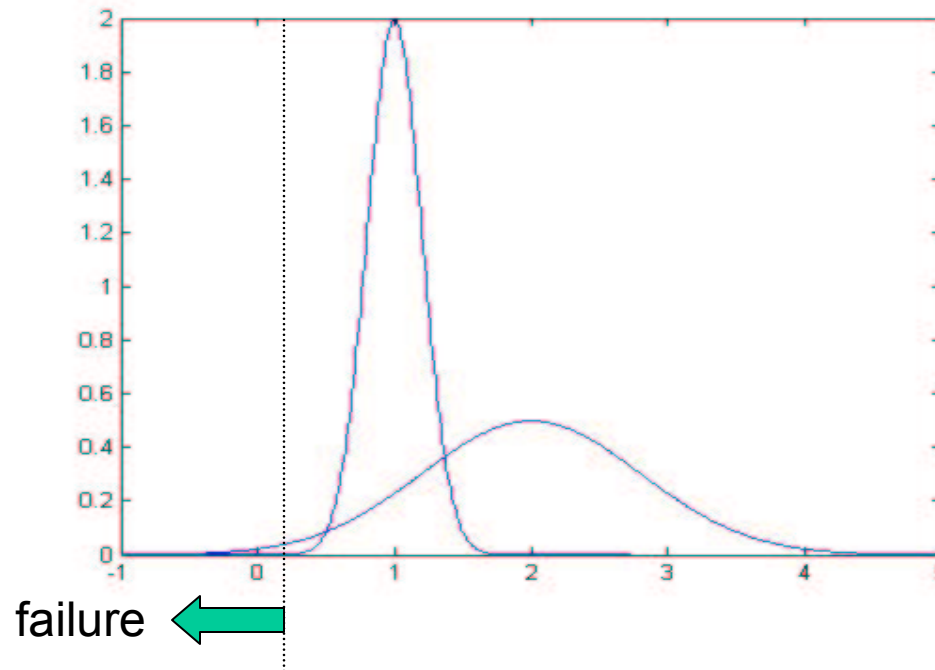
⇒ random distributions of large-scale material parameters

⇒ stochastic failure events, due to rare fluctuations

Why probabilistic?

Importance of spatially varying behavior highlighted
 Random material properties lead to random behavior -
 predicting scatter is critical to reliability estimates

Probability
density
function of
strength



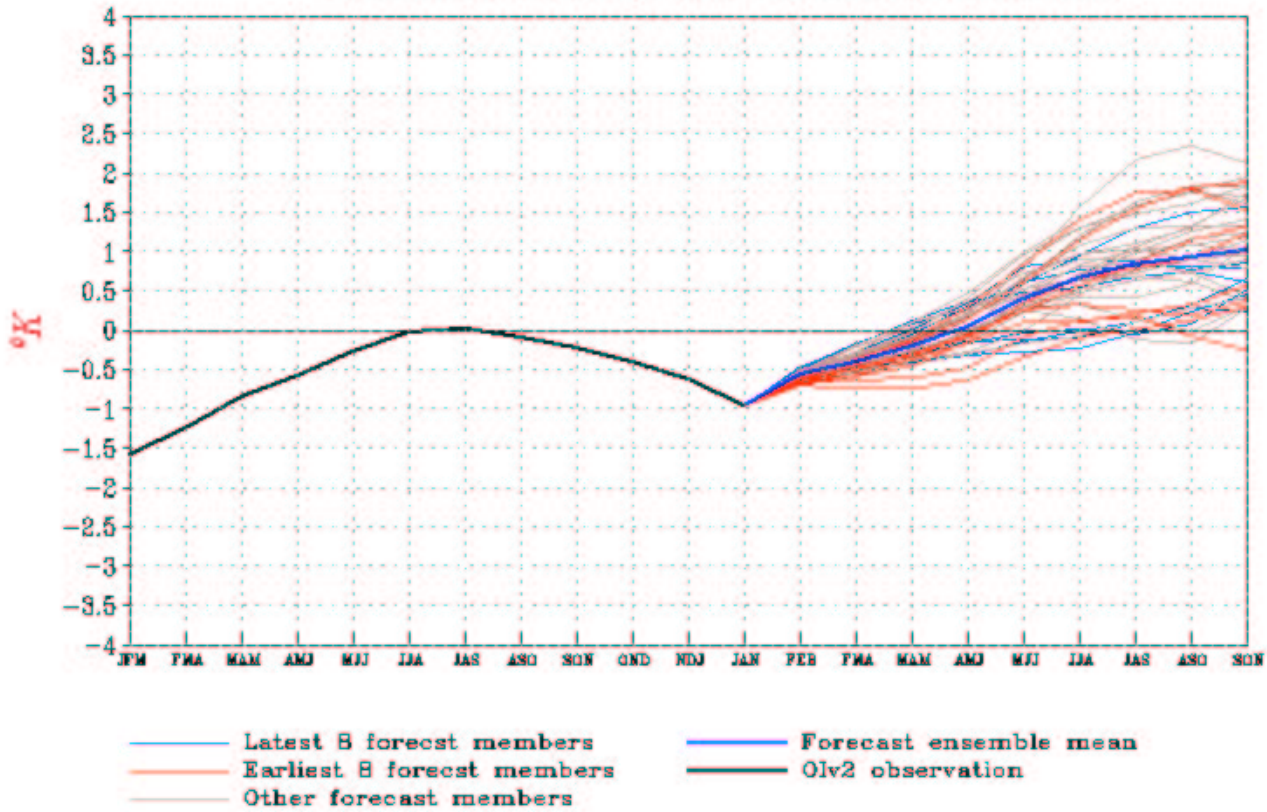
*Higher mean
& higher
variance lead
to more likely
failure than
lower mean &
lower
variance*



NWS/NCEP

Last update: Tue Feb 17 2009
Initial conditions: 6Feb2009-15Feb2009

Forecast Niño3.4 SST anomalies from CFS



* The problem of climate forecasting (discussed in 560.900 by T. Haine) requires a probabilistic approach, because predictions are inherently uncertain. Operational forecasts use ensembles of model runs. For example, see the documentation on the NCEP Climate Forecast System (CFS) at

<http://cfs.ncep.noaa.gov>

Useful forecasts must provide information about ensemble variation or uncertainty in the forecast.

* The energy flux in turbulent flow is given by the deformation work of the large-scale strain rate \bar{S}_L acting against the small-scale stress:

$$\Pi_L = -\bar{S}_L : \tau_L$$

Energy cascade is not strictly forward. Instead, energy flux has a random distribution, including negative values (a "backscatter"), although the average value is positive (from large to small scales).

Stochastic backscatter leads to greater variability and larger fluctuations in the resolved scales, important for some applications (e.g. prediction in turbulent geophysical flows.)

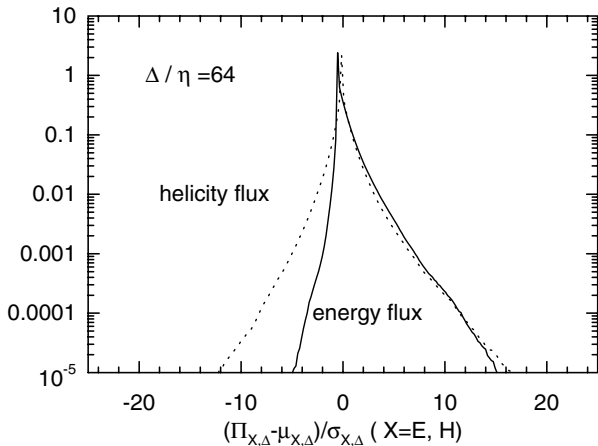


FIG. 4. 1-point PDF's of energy and helicity fluxes [$\mu_{X,\Delta} = \langle \Pi_{X,\Delta} \rangle$, $\sigma_{X,\Delta} = \sqrt{\langle (\Pi_{X,\Delta})^2 \rangle - \langle \Pi_{X,\Delta} \rangle^2}$].