Assignment

- For Feb 11 (This Week)
  - Read: A&L, Chapter 2 (Basic Performance Analysis Concepts)
  - Read: A&L, Chapter 3 (Basic Elements of Modern PF Theory)
- For Feb 18 (Next Week)
  - Read: A&L, Chapter 3 (Basic Elements of Modern PF Theory)
  - Read: A&L, Chapter 4 (Capital Asset Pricing Model and its Application to Performance Measurement)
  - Read: E&G, TBA

Basic Performance Analysis

- Basic Performance Analysis Concepts
  - Performance Measurement – First Stage in Performance Analysis and Management
  - Quantify Return on an Asset/Portfolio
  - AIMR Standard – Requirements/Recommendations
  - First Elements of Performance Evaluation
  - References to be used to Compare Returns: Benchmarks and Peer Groups
  - Risk

- Return Calculation
  - Portfolio Return – Basic Formula w/Capital Flow
    - Three methods
      - Internal Rate of Return (IRR)
      - Capital-Weighted Rate of Return (CWR)
      - Time-Weighted Rate of Return (TWR)
Basic Performance Analysis

- Return Calculation
  - Portfolio Return – Basic Formula w/Capital Flow
    - Capital-Weighted Rate of Return (CWR), \( R_{CWR} \)
      \[
      R_{CWR} = \frac{V_T - V_0 - \sum_{i=1}^{n} C_i}{V_0 + \sum_{i=1}^{n} \frac{T - t_i}{T} C_i}
      \]
      where
      - \( V_T \) is the value of the PF at the end of the period
      - \( V_0 \) is the value of the PF at the beginning of the period
      - \( C_i \) is the \( i \)th capital/cash flow for the PF at \( t_i \)
        (pos if a contribution/inflow & neg if a withdrawal/outflow)
  - Rationale
    \[
    R_{CWR} = \left( \prod_{i=1}^{n} \frac{V_{t_i + C_{t_i}}}{V_{t_i}} \right)^{1/T} - 1
    \]
    - The return for the whole period is geometric mean

- Return Calculation
  - Portfolio Return – Basic Formula w/Capital Flow
    - Internal Rate of Return (IRR), \( R_I \)
      \[
      V_0 + \sum_{i=1}^{n} \frac{C_i}{(1 + R_I)^{T - t_i}} = V_T
      \]
    - Rationale
      \[
      (1 + R_I)^T V_0 + \sum_{i=1}^{n} (1 + R_I)^{T - t_i} C_i = V_T
      \]
      - Features
        - Iterative (a challenge to determine on paper)
        - Inflows invested in the PF going forward

- Return Calculation
  - Portfolio Return – Basic Formula w/Capital Flow
    - Time Weighted Rate of Return (TWR), \( R_{TWR} \)
      - Idea: Break down period \( T \) into elementary sub-periods (cash flows); using the notation as before
        - For each sub-period
          \[
          R_{s_i} = \frac{V_{t_i + C_{t_i}}}{V_{t_i}} - 1
          \]
        - The return for the whole period is geometric mean
          \[
          R_{TWR} = \left[ \prod_{i=1}^{n} \left( 1 + R_{s_i} \right) \right]^{1/T} - 1
          \]
Basic Performance Analysis

- Return Calculation
  - Portfolio Return – Basic Formula w/Capital Flow
  - Time Weighted Rate of Return (TWR), $R_{TWR}$
  - Rationale
    \[(1 + R_{TWR})^T = \prod_{i=1}^{n} (1 + R_i)\]
  - Features
    - To implement, need to know amount and timing of cash as well as value of PF at each date
    - In practice, cash is assumed to occur at month end instead of exact dates; continuous version helps …

\[e^{r_{TWR}} = \text{exp} \left[ \ln \left( \frac{V_T}{V_0} \right) + \sum_{i=1}^{n} \ln \left( \frac{V_i}{V_{i-1} + C_{i-1}} \right) \right] = \frac{V_T}{V_0} \prod_{i=1}^{n} \left( \frac{V_i}{V_{i-1} + C_{i-1}} \right) = \prod_{i=1}^{n} \left( \frac{V_i}{V_{i-1} + C_{i-1}} \right)\]

Basic Performance Analysis

- Return Calculation
  - Portfolio Return – Basic Formula w/Capital Flow
  - Continuous TWR, $r_{TWR}$
    \[r_{TWR} = \frac{1}{T} \sum_{i=1}^{n} \ln \left( \frac{V_i}{V_{i-1} + C_{i-1}} \right) = \frac{1}{T} \left[ \ln(V_T) - \ln(V_0) + \ln(V_{i-1}) - \ln(V_i + C_i) + \cdots + \ln(V_1) - \ln(V_0 + C_0) \right] \]
    \[= \frac{1}{T} \left[ \ln \left( \frac{V_T}{V_0} \right) + \sum_{i=1}^{n} \ln \left( \frac{V_i}{V_{i-1} + C_{i-1}} \right) \right] \]

Basic Performance Analysis

- Return Calculation
  - Portfolio Return – Basic Formula w/Capital Flow
  - Three methods: IRR, CWR, TWR
  - Comparison
    - TWR allows manager to be evaluated separately from movement of capital – best for evaluating manager
    - CWR allows for total performance of fund to be measured
    - IRR more precise than CWR when there is a significant number of capital flows of different sizes
    - Frequent evaluations of PF reduce impact of capital flow
      - Daily is best
Basic Performance Analysis

- Choice of Return Measure – CWR vs. TWR
  - Consider the Fund with NAV as follows

Basic Performance Analysis

- Choice of Return Measure – CWR vs. TWR
  - There are two investors
    - Each start the year purchasing 100 units at the end of Dec
    - Investor 1 makes 2 subsequent purchases of 100 units each at the end of May (NAV = 14) and end of August (NAV = 15)
    - Investor 2 makes 2 subsequent purchases of 100 units each at the end of April (NAV = 8) and end of September (NAV = 9)
  - A summary of their activity is shown as follows

Basic Performance Analysis

- Choice of Return Measure – CWR vs. TWR
  - In each case according to the TWR they obtain the same return as the fund

Basic Performance Analysis

- Choice of Return Measure – CWR vs. TWR
  - Which makes no sense to either investor, even if it makes perfect sense to the manager
    - IRR₁ = -24.86% & IRR₂ = 35.16%
  - The investors don’t see credit or blame for their choices in capital timing
Basic Performance Analysis

- **Choice of Return Measure – CWR vs. TWR**
  - Consider a second situation
    - A manager makes allocation between three asset classes and generates the following return
      - This seems to be counterintuitive; how can a portfolio exceed the return of its components
    - Until we consider capital flows

<table>
<thead>
<tr>
<th></th>
<th>Equities</th>
<th>Bonds</th>
<th>Cash</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>BOR</td>
<td>2.32%</td>
<td>-0.38%</td>
<td>3.24%</td>
<td>6.01%</td>
</tr>
</tbody>
</table>

Basic Performance Analysis

- **Choice of Return Measure – CWR vs. TWR**
  - Consider a second situation
  - Until we consider capital flows

<table>
<thead>
<tr>
<th></th>
<th>Equities</th>
<th>Bonds</th>
<th>Cash</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>BMV</td>
<td>10,000.00</td>
<td>10,000.00</td>
<td>80,000.00</td>
<td>100,000.00</td>
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<tr>
<td>BOR</td>
<td>-2.00%</td>
<td>-1.50%</td>
<td>0.80%</td>
<td>0.29%</td>
</tr>
<tr>
<td>EMV 1Q</td>
<td>9,900.00</td>
<td>9,930.00</td>
<td>80,640.00</td>
<td>100,270.00</td>
</tr>
<tr>
<td>BOR</td>
<td>-3.00%</td>
<td>-2.00%</td>
<td>0.80%</td>
<td>0.10%</td>
</tr>
<tr>
<td>EMV 2Q</td>
<td>9,930.00</td>
<td>9,630.00</td>
<td>81,385.12</td>
<td>100,445.12</td>
</tr>
</tbody>
</table>

- **Equities**
- **Bonds**
- **Cash**
- **Total**

- The key to choosing CWR or TWR is to identify who controls the flow of capital
  - The return should be calculated associated with control of the flow of capital
    - In the first example, the manager did not control the capital flow
      - To describe his performance – use TWR
    - In the second example, the investor controlled the capital flow
      - To describe the performance of his account – use CWR
  - In the second example, the managers return in the asset classes needed to reflect his decision on allocation – thus asset class returns are higher
Basic Performance Analysis

- Return Calculation

  - **HW An Example (pg 32)**

    - **CWR**
      \[
      R_{CWR} = \frac{V_T - V_0 - \sum_{i=1}^{T} C_i}{V_0} \approx 45.28\%
      \]

    - Converting to a standard reference period rate
      \[
      (1 + R_{CWR}) = (1 + R_{CWR}) = (1 + 0.4528) \Rightarrow R_{CWR} = 13.26\%
      \]

    - **IRR** = 13.17%
    - **TWR**
      \[
      R_{TWR} = \left[ \prod_{t=1}^{T} \left( \frac{V_t}{V_{t-1}} \right) \right]^{1/T} - 1 = 14.26\%
      \]
    - **TWR in continuous time** = 13.33%

Basic Performance Analysis

- Return Calculation

  - **International Investment**

    - Up to now, assumption of single currency
    - Exchange rate definitions
      - Forward premium (or discount), \( f : \)
        \[
        \text{forward premium (discount)} = \left( \text{forward x-rate} - \text{spot x-rate} \right) / \text{spot x-rate}
        \]
      - \( f \): spot price of currency in reference ($), also \( r_{ref} \): ref ($) risk free rate, currency risk free rate
    - \( F_r \): fwd price of currency in reference ($), also \( F^{Q,R}_r \)
      \[\text{Forward premium (discount)} = ( f \times \text{x-rate} - \text{spot x-rate}) / \text{spot x-rate} \]

    - Remember:
      \[
      F_r = S_r \times (1 + \text{fwd premium}) \quad \text{where}
      \]
      \[S_r \quad \text{spot price of currency in reference ($), also } F^{Q,R}_r \]
      \[r_{ref}, r_{fwd} \quad \text{ref ($) risk free rate, currency risk free rate} \]

Basic Performance Analysis

- Return Calculation

  - **International Investment**

    - Evolution of the spot exchange rate in percentage terms is called the “currency return”
    - The variable TC is used to refer to this quantity
    - In an international portfolio, the currency return needs to be “bifurcated” from the asset return (when the asset is priced in currency)
      - Return is expressed as the sum of the return on the asset and the currency return

Basic Performance Analysis

- Return Calculation

  - **International Investment**

    - Extension of return models (arithmetic & logarithmic) provides (asset – currency) bifurcation of return
      - Starting with the price representation in the Reference currency of asset at time \( i \) in terms of the Quotation currency:
        \[
        P_i^{Q,R} = p_i^{Q,R} d_i^{Q,R} \quad \text{&} \quad d_i^{Q,R} : \text{Price of quote currency in the reference currency}
        \]
      - Allows the development of the arithmetic return:
        \[
        (1 + R_i^{Q,R}) = \frac{P_i^{Q,R}}{P_{i-1}^{Q,R}} = \left( \frac{p_i^{Q,R}}{p_{i-1}^{Q,R}} \right) \left( \frac{d_i^{Q,R}}{d_{i-1}^{Q,R}} \right) = (1 + R_i^{Q,R}) (1 + TC_i^{Q,R})
        \]
        \[
        = 1 + R_i^{Q,R} + TC_i^{Q,R} + R_i^{Q,R} TC_i^{Q,R} \implies 1 + R_i^{Q,R} + TC_i^{Q,R}
        \]
      - \( R_i^{Q,R} \approx R_i^{Q,R} + TC_i^{Q,R} \)
Basic Performance Analysis

• Return Calculation
  • International Investment
    • Similarly for the Logarithmic Return we have the exact equality
      \[ R_b^L = \ln \left( \frac{P_{i+1}^o}{P_{i+1}^d} \right) = \ln \left( \frac{P_i^o}{P_i^d} \right) + \ln \left( \frac{d_{i+1}^o}{d_{i+1}^d} \right) = R_b^O + TC_{t+1}^{QR} \]
    • For the portfolio, we also have the expected ref result
      \[ V_r^L = \sum_{i=1}^n n_i P_i^d = \sum_{i=1}^n n_i P_i^o d_i^O \]
      & if quote = ref currency, \( d_i^O = 1 \)
      \[ R_p^L = \sum_{i=1}^n x_i R_i^d + \sum_{i=1}^n x_i TC_i^{QR} + \sum_{i=1}^n x_i R_i^{QR} \]: arithmetic &
      \[ R_p^L \equiv \sum_{i=1}^n x_i R_i^d + \sum_{i=1}^n x_i TC_i^{QR} + \sum_{i=1}^n x_i TC_i^{QR} \]: logarithmic

1.25

Basic Performance Analysis

• Return Calculation
  • International Investment
    • So return on an asset hedged w/fwd for currency risk is
      \[ (1 + R_f^L) = (1 + R_b^O) \left( 1 + T C_{t+1}^{QR} \right) + h \left( T C_{t+1}^{QR} - f_u \right) \]
      where the hedge ratio \( h \in [-1,0] \)
    • Expanding (setting aside the cross term and add/sub \( f_u \) )
      \[ (1 + R_f^L) = (1 + R_b^O + T C_{t+1}^{QR} + R_u^{QR} T C_{t+1}^{QR}) h \left( T C_{t+1}^{QR} - f_u \right) \]
    • The sum of the completely hedged position plus the unhedged proportion of the exchange exposure

1.26

Basic Performance Analysis

• Return Calculation
  • International Investment
    • For bifurcation when hedge instruments are used
      • Note the return, \( R_{FC} \), for a forward exchange contract
      \[ R_{FC} = \frac{S_t - F_0}{F_0} = \text{diff between currency return & fwd premium return} \]
      \[ = 1 + T C_{t+1}^{QR} - (1 + f_u) \text{ where } (1 + f_u) \text{ is the forward premium on } [0,7] \]
      \[ = T C_{t+1}^{QR} - f_u \]
      • This return is sometimes referred to as the “forward surprise”

1.27

Basic Performance Analysis

• Return Calculation
  • International Investment
    • Similarly for the hedged portfolio of \( n \) assets
      \[ R_{FC} = \sum_{i=1}^n x_i R_i^O + \sum_{i=1}^n x_i T C_{t+1}^{QR} + \sum_{j=1}^m h_j \left( T C_{t+1}^{QR} - f_{j+1} \right) \]
      where \( h_j \) is the hedge ratio for each of \( m \) currencies

1.28
Basic Performance Analysis

- Return Calculation
  - Using Derivatives – Consider the simple case of a call option to appreciate what can be done
    - Simply, the contribution of the derivative is the performance of the underlying alone differed from the performance including the derivative; we can say more about whence the derivative performance comes
    - The price performance of the call can be written \( C_t - C_0 \)
      - Now suppose the theoretical value of the call at \( t \) is \( V_0 \)
      - Then it is possible to break the performance into 2 terms
        \[
        C_t - C_0 = (V_0 - C_0) + (C_t - V_0)
        \]

- Return Calculation
  - Using Derivatives – Consider the simple case of a call option to appreciate what can be done
    \[
    C_t - C_0 = (V_0 - C_0) + (C_t - V_0)
    \]
    - The first term measures the theoretical vs. quoted price based on the spot price of the underlying on date of purchase
    - 2nd term measures the differential between the current quote and the initial equilibrium price
    - The first can measure the ability of the manager to pick undervalued options
    - The second measures the ability to select options with undervalued underlying assets (as we shall see)
    - Are the outcomes from luck or skill?

- Return Calculation
  - Using Derivatives – Consider the simple case of a call option to appreciate what can be done
    \[
    C_t - C_0 = (V_0 - C_0) + (C_t - V_0)
    \]
    - Each term can be broken down again
      - The first can be broken down to a volatility profit and a formula profit
        \[
        V_0 - C_0 = \left( C_0(s) - C_0 \right) + \left( V_0 - C_0(s) \right)
        \]
        - Note 3 formulas: the true, the market, and the benchmark
        - We use the benchmark (market) with the implied vol. at \( t \) vs. the real vol. \( s \) to determine the vol. profit
        - The formula profit is found using the benchmark (true) with the real vol.

- Return Calculation
  - Using Derivatives – Consider the simple case of a call option to appreciate what can be done
    \[
    C_t - C_0 = (V_0 - C_0) + (C_t - V_0)
    \]
    - Each term can be broken down again
      - The second term can be broken down into the profit from asset undervaluation and one due only to the option overlay
        - The asset profit is measured relative to a benchmark strategy – a forward contract on the underlying
        - Assuming the market is efficient and risk neutral, this profit is zero

1.30

1.31

1.32

1.33
Basic Performance Analysis

- Return Calculation
  - Using Derivatives – Consider the simple case of a call option to appreciate what can be done
    \[ C_t - C_0 = (V_0 - C_0) + (C_t - V_0) \]
    - Each term can be broken down again
    - The second term can be broken down into the profit from asset undervaluation and one due only to the option overlay
    - Finally, the profit from using an option vs. a forward
    \[ C_t - V_0 - (S_t - S_0 e^{(r-d)t}) \]
  - So we have
    \[ C_t - C_0 = (C_0(s) - C_0) + (V_0 - C_0(s)) + (S_t - S_0 e^{(r-d)t}) + \left[ C_t - V_0 - (S_t - S_0 e^{(r-d)t}) \right] \]

1.34

Basic Performance Analysis

- Return Calculation
  - Using Derivatives – Consider the simple case of a call option to appreciate what can be done
    \[ C_t - C_0 = (C_0(s) - C_0) + (V_0 - C_0(s)) + \left[ C_t - V - (S_t - S_0 e^{(r-d)t}) \right] \]
  - Vol. term + formula term + asset term + option alone
  - If options are valued efficiently, the sum of the first two is zero (on average), irrespective of the benchmark formula being close to the one used by the market
  - If options are valued efficiently and the benchmark coincides with market formula, both terms are zero.

1.35

Basic Performance Analysis

- Return Calculation
  - Using Derivatives – Consider the simple case of a call option to appreciate what can be done
    \[ C_t - C_0 = (C_0(s) - C_0) + (V_0 - C_0(s)) + \left[ C_t - V - (S_t - S_0 e^{(r-d)t}) \right] \]
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1.35

Basic Performance Analysis

- Return Calculation – The GIPS
  - Performance
    - Must be on a total return basis using time weighted rate of return
    - Best is to value on a daily basis and combine results geometrically
    - Calculated at least quarterly, but recommends monthly (net of transaction costs and including return on cash)

1.36

Basic Performance Analysis

- Return Calculation – The GIPS
  - Performance
    - Must be on a total return basis using time weighted rate of return
    - Best is to value on a daily basis and combine results geometrically
    - Calculated at least quarterly, but recommends monthly (net of transaction costs and including return on cash)
Basic Performance Analysis

1.38

- Relative Return Calculation
  - Better than absolute return, it is more relevant to measure performance in relation to a reference
  - Can thereby highlight the additional share of return that comes from the investment strategy used and the managers skill
  - The reference can be
    - A Benchmark, or
    - A group of portfolios with the same characteristics, called a Peer Group

1.39

- Relative Return Calculation
  - Benchmarks
    - A benchmark is simply a reference portfolio
    - It must be chosen to reflect
      - Diversity of Assets eligible for the "measured" portfolio
      - The Investment Strategy
      - The same Calculation Rules as the "measured" portfolio (the portfolio being evaluated)
        - Especially, wrt to dividend/cash flow and reinvestment
  - Types of Benchmarks ...

1.40

- Relative Return Calculation
  - Types of Benchmarks
    - Market Indices
      - Quoted on Exchanges – simple and readily accessible
      - May not sufficiently represent objective or strategy of measured PF
      - Broad (includes less liquid, e.g. Wilshire) vs. More Liquid Indices (more liquid, more restricted, e.g. Dow) vs. Mid-Size
    - Generic Investment Style Indices
      - Developed by Specialized Firms & Allow Different Styles
      - Growth, Value, Small-Cap, etc.
      - Appropriate for managers with a well-defined, "standard" investment style

1.41

- Relative Return Calculation
  - Types of Benchmarks
    - Generic Investment Style Indices (continued)
      - For "non-standard" styles there are benchmarks that better describe a manager's style
      - Difficult to select components for this type of index
      - Is a low P/E stock a value or an undervalued growth stock
      - No Standard – Lots of Competition – Low correlation
      - Frank Russell, S&P-Barra, & others
Basic Performance Analysis

- Relative Return Calculation
  - Types of Benchmarks
    - Sharpe Benchmarks
      - Explains a manager’s style from performing a multiple regression on several specialized indices (asset classes and markets that the manager is in)
    - Normal Portfolios (the trend)
      - Tailor-made for each manager
      - Uses the principle that the PF manager’s returns should be compared with the returns of a reference PF whose structure and composition are as similar as possible to those of the PF being evaluated

1.42

Basic Performance Analysis

- Relative Return Calculation
  - Types of Benchmarks
    - Normal Portfolios (the trend)
      - Two techniques for choosing weightings (more later)
      - Returns-Based: Analysis of the managers PF returns
      - Portfolio-Based: Analysis of style characteristics of the securities that makeup the managers PF
      - Historical Composition (most common for choosing securities)
        - Define an initial universe
        - Choose securities included in the normal PF (managers rules compared to actual history)
      - Choose Weightings (market cap, equal, market cap with breakpoints on capitalization)

1.44

Basic Performance Analysis

- Relative Return Calculation
  - Types of Benchmarks
    - Normal Portfolios (the trend)
      - Portfolio is made up of a set of securities that contains all the securities from which the manager will choose; weighted in the same way as would be expected in the managers PF
      - Objective: a portfolio that obtains an average characterization of the PF to be evaluated
      - Three techniques for choosing securities (to reduce the number from the universe (2); or express as a style)
        - Based on historical composition
        - Based on historical risk/factor exposure
        - Using Style Indices (Sharpe)

1.43

Basic Performance Analysis

- Relative Return Calculation
  - Types of Benchmarks
    - Practical Use of the Benchmark
      - Managers Value-Add is calculated as the difference between the return on the PF being evaluated and the benchmark
      - We will look at precise measures as we go; later in a larger portfolio theory context (what the course is all about)
      - Active Return – the percentage of return that is due to the manager’s decisions
      - Manager’s Skill over a given period of time t is defined as
        \[ R_{pa} = R_{p} - R_{b} \]

1.45
Basic Performance Analysis

- Relative Return Calculation
  - Types of Benchmarks
    - Practical Use of the Benchmark
      \( R_{bu} = R_p - R_b \)
      - The calculation may be made periodically (daily, monthly) and then compounded for the whole period
      - If (sub-period) returns are calculated arithmetically and then compounded geometrically, then the cumulative active return for the whole period is not equivalent to calculating the difference between the cumulative PF return and the cumulative BM return
      - If we have worked with logarithmic returns, the two calculations are equivalent – logarithmic returns allow\(^{1.46}\) simpler formulas

- Peer Groups
  - Distinguish between Universes and Peer Groups
    - Universe is a group of PFs invested in the same market sector
    - Peer Group is a group of managers who invest in the same class of assets or have the same investment style
    - Peer Groups are smaller and more precise\(^{1.48}\)

Basic Performance Analysis

- Relative Return Calculation
  - Types of Benchmarks
    - Practical Use of the Benchmark
      \( R_{bu} = R_p - R_b \)
      - Consider \( T \) sub-periods, then the logarithmic returns for the PF and the BM are written as
        \[ R_{p}^{\text{log}} = \sum_{t=1}^{T} R_{p,t}^{\text{log}} \quad \text{and} \quad R_{b}^{\text{log}} = \sum_{t=1}^{T} R_{b,t}^{\text{log}} \]
        gives
        \[ R_{p}^{\text{log}} - R_{b}^{\text{log}} = \sum_{t=1}^{T} (R_{p,t}^{\text{log}} - R_{b,t}^{\text{log}}) = \sum_{t=1}^{T} R_{p,t}^{\text{log}} - R_{b,t}^{\text{log}} = R_{p}^{\text{log}} - R_{b}^{\text{log}} \]

- Peer Groups
  - Peer Groups are put together by choosing cohort managers – e.g., same style: market cap, growth, value, emerging markets, ...
  - Calculate returns and establish a ranking within the group
  - Comparisons are made between real PFs with transaction costs, taxes, etc. and not indices as theoretical, cost-free PFs
  - Technical ranking considerations\(^{1.49}\)
Basic Performance Analysis

- Relative Return Calculation
  - Portfolio Opportunity Distributions (New)
    - Seeks to combine advantages of benchmarks and peer groups while eliminating their disadvantages
    - Principle: Compare managers result with that achieved by chance
      - Start with the securities the manager is liable to include
      - Generate a large number of PFs representative of the managers style
      - The manager's performance is compared to the result universe
        - Overcomes problem of establishing cohorts; survivorship bias; statistically significant numbers; and luck vs. skill.

Basic Performance Analysis

- Risk
  - Return is (of course) not sufficient to analyze the results of a portfolio – it is necessary to add a measurement of the risk taken in the PF
  - Some simple statistical risk measures – to get started, we expand greatly later
    - Variance, $\sigma_i^2$ (and standard deviation, $\sigma_i$)
      \[
      \sigma_i^2 = \frac{1}{T} \sum_{t=1}^{T} (R_{it} - \bar{R}_i)^2
      \]
      where
      - $R_{it}$ is return on asset $i$ for subperiod $t$
      - $\bar{R}_i$ is mean return on asset $i$ for whole period
      - $T$ is the number of subperiods

Basic Performance Analysis

- Risk
  - Variance, $\sigma_i^2$ (and standard deviation, $\sigma_i$)
    - A good estimate of risk is obtained by using monthly returns over a 3 year period
    - This is the most common risk measure used in theoretical discussions
    - Has the disadvantage of symmetry
      - Above average returns and below average returns are equally weighted – investors don’t see it that way, though
Basic Performance Analysis

Risk

- Semi-Variance
  - A natural extension for variance in analyzing PF risk is
  \[ \frac{1}{T} \sum_{t=1}^{T} (R_t - \bar{R})^2 \]
  - In an analogy to the relationship between the standard deviation and variance, we define a measure called the “downside risk” as the square root of the semi-variance
  - If the distribution of returns is symmetrical (e.g., Gaussian) then the semi-variance is equal to half the variance
  - More interesting when returns are skewed – non-linear risk exposures
  - Difficult to estimate from historical returns – not stable over time

- Lower Partial Moments (LPM) of degree \( n \) for asset \( i \)
  \[ LPM_n = \frac{1}{T} \sum_{t=1}^{T} \max (0, h - R_t) \]
  - When \( n=2 \) we find the semi-variance expression by taking the mean return as the target return; we can use other targets too
  - \( n \) allows the investor’s risk aversion to be represented
    - \( n<1 \): investor is risk seeking
    - \( n=1 \): investor is risk neutral
    - \( n>1 \): investor is risk averse (more so as \( n \) increases)

Asset risk linkage with multiple assets

- Covariance
  \[ \sigma_{ij} = \frac{1}{T} \sum_{t=1}^{T} (R_t - \bar{R}_i)(R_t - \bar{R}_j) \]
- Normalized version (w/r standard deviations) is correlation
  \[ \rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} \]

- Other risk measures
  - Variation interval (highest to lowest)
  \[ \max R_t - \min R_t \]
  - Absolute mean deviation
  \[ \frac{1}{T} \sum_{t=1}^{T} |R_t - E(R)| \]
  - Probability of negative return – the proportion of negative asset returns over a given period

Two examples return vs. risk & diversification

- Consider the case of bonds & stocks
  - Let us conclude from this and from analyst forecasts:
    - Bond: \( R_{B} = 12.5\% \)  \( \sigma_{B} = 14.9\% \)  \( \rho_{B,P} = 0.45 \)
    - Stock: \( R_{S} = 6\% \)  \( \sigma_{S} = 4.8\% \)
Basic Performance Analysis

- **Risk**
  - Two examples return vs. risk & diversification
    - Using the standard formulas in forming the PF of stocks and bonds, we get

<table>
<thead>
<tr>
<th>Proportion Stocks</th>
<th>Proportion Bonds</th>
<th>Mean Return</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>12.5</td>
<td>14.90</td>
</tr>
<tr>
<td>0.9</td>
<td>0.1</td>
<td>11.85</td>
<td>13.63</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2</td>
<td>11.2</td>
<td>12.38</td>
</tr>
<tr>
<td>0.7</td>
<td>0.3</td>
<td>10.55</td>
<td>11.15</td>
</tr>
<tr>
<td>0.6</td>
<td>0.4</td>
<td>9.9</td>
<td>9.95</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>9.25</td>
<td>8.80</td>
</tr>
<tr>
<td>0.4</td>
<td>0.6</td>
<td>8.6</td>
<td>7.70</td>
</tr>
<tr>
<td>0.3</td>
<td>0.7</td>
<td>7.95</td>
<td>6.69</td>
</tr>
<tr>
<td>0.2</td>
<td>0.8</td>
<td>7.3</td>
<td>5.82</td>
</tr>
<tr>
<td>0.1</td>
<td>0.9</td>
<td>6.65</td>
<td>5.16</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>6</td>
<td>4.80</td>
</tr>
</tbody>
</table>

Now consider the case of domestic & foreign stocks.

- We conclude from historical data and analysts forecasts that
  - \( \bar{R}_{SK} = 12.5\% \), \( \sigma_{SK} = 14.9\% \), \( \rho_{SK,INT} = 0.33 \)
  - \( \bar{R}_{INT} = 10.5\% \), \( \sigma_{INT} = 14.9\% \)
Basic Performance Analysis

- **Risk**
  - Two examples return vs. risk & diversification
    - Graphically

![Graph](1.62)

Basic Performance Analysis

- **Risk**
  - Fixed Income – it’s duration and convexity: about which more later
  - International Assets and Currency Risk
    - We have already seen how to bifurcate returns
    - To measure risk, we apply the above
      \[
      \text{var}(R_i) = \text{var}(R_i^0) + \text{var}(\mathit{TC}_i^{Q,R}) + 2 \text{cov}(R_i^0, \mathit{TC}_i^{Q,R})
      \]

\[
\sigma_i^2 = \sigma_i^0 + \sigma_i^{Q,R} + 2 \rho_{i,QC} \sigma_i^0 \sigma_i^{Q,R}
\]

1.63

Basic Performance Analysis

- **Risk**
  - International Assets and Currency Risk
    - Hedging currency exposure ($H = 1 + h$, from earlier)
      \[
      \left( \sigma_i^2 \right)^2 = \left( \sigma_i^0 \right)^2 + H^2 \left( \sigma_i^{Q,R} \right)^2 + 2H \rho_{i,QC} \sigma_i^0 \sigma_i^{Q,R}
      \]
    - If $H=0$, we are hedged; if $H=1$, we are un-hedged
    - Extending to a PF of domestic asset and an Intl asset
      - Cross terms get to be a nuisance (as above with values of $H$)
        \[
        \left( \sigma_p^2 \right)^2 = \sum_i \sigma_i^2 + \sum_i \sigma_i^{Q,R} + H^2 \sum_i \sigma_i^{Q,R} + 2 \sum_i \sigma_i^{Q,R} + 2H \sum_i \left( x_i \sigma_{d,JC} + x_i \sigma_{j,JC} \right)
        \]
      - The variance of the un-hedged PF will be greater than that of the hedged PF if
        \[
        \sum_i x_i^2 \sigma_{JC} + 2 \sum_i x_i \left( x_i \sigma_{d,JC} + x_i \sigma_{j,JC} \right) > 0
        \]

1.64

Basic Performance Analysis

- **Risk**
  - Have looked at specific risk indicators by instrument: used by traders & sector specialists
    - Variance & Beta for Stocks
    - Duration & Convexity for Bonds
    - Delta & Vega for Derivatives
    - To roll up risk measures & consider risk more broadly through a “simplified” representation
      - Use Value-at-Risk (VaR)
Basic Performance Analysis

- Risk – The VaR Measure
  - To provide a single number to characterize the risk for loss in a portfolio
  - Complements trader risk measures for assessing risk over the whole portfolio

1.66

- Risk – The VaR Measure
  - Let the variable $V$ be the VaR of the portfolio
  - It is a function of two parameters
    - The time horizon (N days)
    - The confidence level (X %)
  - The loss level, $V$, over N days that we are X % certain will not be exceeded

1.67

- Risk – The VaR Measure
  - We usually use the Standardized VaR terms from regulators (for market risk)
    - Confidence level of 99% (loss won’t occur more than 1 time in 100)
    - 10-day period is the usual term for a market reversal
    - VaR is the 1% quantile of the PF return pdf over 10 days

1.68

- Risk – The VaR Measure
  - If N days is the time horizon & X% the confidence,
    - VaR is the loss corresponding to the (100-X)th percentile of the distribution of the change in the value of the portfolio over the next N days

1.69
Basic Performance Analysis

Risk – The VaR Measure

- The simplest assumption is that daily gains/losses are normally distributed and independent.
- It is then easy to calculate VaR from the standard deviation $\sigma$ of gains/losses (1-day VaR=$2.33 \times \sigma$)
  - Since $N(-2.33)=0.01$ or $N(2.33)=0.99$
  - The $N$-day VaR equals $\sqrt{N}$ times the one-day VaR
  - Regulators allow banks to calculate the 10 day VaR as $\sqrt{10}$ times the one-day VaR

Risk – The VaR Measure: Historical Simulation

- Create a database of the daily movements in all market variables.
- The first simulation trial assumes that the percentage changes in all market variables are as on the first day.
- The second simulation trial assumes that the percentage changes in all market variables are as on the second day.
- and so on

Basic Performance Analysis

- Historical Simulation
- Model Building (Variance – Covariance)
  - Linear Model
  - Quadratic Model – when gamma is significant
- Interest Rates
- Monte Carlo Simulation
- Stress Testing & Back Testing
- Principal Component Analysis
1.74

If we are interested in the 1-percentile point of the distribution of changes in PF value, estimate this as the 5th worst number in the last column of Table 9.2.

The 10-day VaR for a 99% confidence level is usually calculated as $\sqrt{10} \times \text{that 5th worst number}$.

## Basic Performance Analysis

### Table 9.1 Data for VaR historical simulation calculation.

<table>
<thead>
<tr>
<th>Day</th>
<th>Market variable 1</th>
<th>Market variable 2</th>
<th>Market variable 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20.33</td>
<td>0.1132</td>
<td>-95.37</td>
</tr>
<tr>
<td>1</td>
<td>20.78</td>
<td>0.1199</td>
<td>64.91</td>
</tr>
<tr>
<td>2</td>
<td>21.44</td>
<td>0.1162</td>
<td>65.02</td>
</tr>
<tr>
<td>3</td>
<td>20.97</td>
<td>0.1284</td>
<td>64.99</td>
</tr>
<tr>
<td>498</td>
<td>21.72</td>
<td>0.1312</td>
<td>62.22</td>
</tr>
<tr>
<td>499</td>
<td>21.73</td>
<td>0.1323</td>
<td>61.99</td>
</tr>
<tr>
<td>500</td>
<td>21.85</td>
<td>0.1345</td>
<td>62.10</td>
</tr>
</tbody>
</table>

### Table 9.2 Simulation generated for tomorrow (Day 501) using data in Table 9.1.

<table>
<thead>
<tr>
<th>Scenario number</th>
<th>Market variable 1</th>
<th>Market variable 2</th>
<th>Market variable 3</th>
<th>Portfolio value ($ million)</th>
<th>Change in value ($ million)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26.42</td>
<td>0.1575</td>
<td>-94.66</td>
<td>23.71</td>
<td>0.21</td>
</tr>
<tr>
<td>2</td>
<td>26.67</td>
<td>0.1446</td>
<td>62.21</td>
<td>23.12</td>
<td>-0.38</td>
</tr>
<tr>
<td>3</td>
<td>25.28</td>
<td>0.1388</td>
<td>61.99</td>
<td>22.94</td>
<td>-0.96</td>
</tr>
<tr>
<td>498</td>
<td>25.31</td>
<td>0.1398</td>
<td>61.87</td>
<td>23.65</td>
<td>0.13</td>
</tr>
<tr>
<td>499</td>
<td>25.32</td>
<td>0.1363</td>
<td>62.21</td>
<td>22.87</td>
<td>-0.63</td>
</tr>
<tr>
<td>500</td>
<td>25.95</td>
<td>0.1363</td>
<td>62.21</td>
<td>22.87</td>
<td>-0.63</td>
</tr>
</tbody>
</table>

1.75

### Basic Performance Analysis

- **Risk – The VaR Measure: Model Building**
  - The main alternative to historical simulation is to make assumptions about the probability distribution of returns on the market variables and calculate the probability distribution of the change in the value of the portfolio analytically.
  - This is known as the model building approach or the variance-covariance approach.

1.76

### Basic Performance Analysis

- **Risk – The VaR Measure: Model Building**
  - **A Detail – Reminder about Volatility**
    - In option pricing we measure volatility “per year”.
    - In VaR calculations we measure volatility “per day”:
      $$\sigma_{\text{day}} = \frac{\sigma_{\text{year}}}{\sqrt{252}}$$
    - Strictly speaking we should define $\sigma_{\text{day}}$ as the standard deviation of the continuously compounded return in one day.
    - In practice we assume that it is the standard deviation of the percentage change in one day.

1.77

### Basic Performance Analysis

- **Risk – The VaR Measure: Model Building**
  - **We have a position worth $10 million in Microsoft shares**
  - **The volatility of Microsoft is 2% per day (about 32% per year)**
  - **We use $N=10$ and $\lambda=99$**
  - **The standard deviation of the change in the portfolio in 1 day is $200,000$**
    - **2% of $10$ million**
    - **The standard deviation of the change in 10 days is $200,000 \sqrt{10} = $632,456**
Basic Performance Analysis

- Risk – The VaR Measure: Model Building
  - We assume that the expected change in the value of the portfolio is zero (This is OK for short time periods)
  - We assume that the change in the value of the portfolio is normally distributed
  - Since $N(-2.33) = 0.01$, the (10-day, 99%) VaR is $2.33 \times 632,456 = $1,473,621

Basic Performance Analysis

- Risk – The VaR Measure: Model Building
  - Consider a position of $5 million in AT&T
  - The daily volatility of AT&T is 1% (approx 16% per year)
  - The S.D per 10 days is $50,000\sqrt{10} = $158,144
  - The VaR is $158,144 \times 2.33 = $368,405$

Basic Performance Analysis

- Risk – The VaR Measure: Model Building
  - Now consider a portfolio consisting of both Microsoft (X) and AT&T (Y)
  - Suppose that the correlation between the returns is 0.3
  - So a portfolio of X and Y has standard deviation
    $\sigma_{X,Y} = \sqrt{\sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y}$
  - In this case $\sigma_X = 200,000$ and $\sigma_Y = 50,000$ and $\rho = 0.3$.
    The standard deviation of the change in the portfolio value in one day is therefore $220,227$

Basic Performance Analysis

- Risk – The VaR Measure: Model Building
  - The 10-day 99% VaR for the portfolio is $220,227 \times \sqrt{10} \times 2.33 = $1,622,657
  - The "benefit of diversification" is $(1,473,621 + 368,405) - 1,622,657 = $219,369$
  - What is the incremental effect of the AT&T holding on VaR?
    - Driven by the correlation
      - At 0 : $1,842,026$ (increases by 368,405)
      - At 0.3 : $1,622,657$ (increases by 219,405)
Basic Performance Analysis

- Risk – The VaR Measure: Model Building
  - For the general Linear Model for VaR, assume:
    - The daily change in the value of a portfolio is linear in the daily returns from market variables
    - The returns from the market variables are normally distributed
      \[ \Delta P = \sum \alpha_i \Delta x_i \] where \( \Delta x_i \) is % change in value of asset \( i \)
      \[ \sigma^2 = \sum \sum \alpha_i \alpha_j \sigma_i \sigma_j \rho_{ij} + 2 \sum \alpha_i \sigma_i^2 \] where \( \sigma_i \) is the volatility of variable \( i \)
      and \( \sigma_P \) is the portfolio's standard deviation.