Some rules.

Generally, if you find that a stated problem is only solvable under additional hypotheses, state the hypotheses required, and if possible, give an counterexample to show that the additional hypotheses are indeed required.

In your writeup of solutions, any materials you quote verbatim should have quotation marks and a reference.

If you get help with a problem, you should acknowledge this in your writeup.

1. (a) Suppose $Y_1, \ldots, Y_n$ are iid $N(0, 1)$, and $c_1, \ldots, c_n$ are constants satisfying $\sum_{i=1}^n c_i = 0$ and $\sum_{i=1}^n c_i^2 = 1$. Show that

$$\sqrt{n - 2} \frac{\sum_{i=1}^n c_i Y_i}{\sqrt{\sum_{i=1}^n Y_i^2 - n\bar{Y}^2 - (\sum_{i=1}^n c_i Y_i)^2}}$$

has a $t_{n-2}$ distribution. Hint: There is an orthogonal matrix whose rows are $(1/\sqrt{n}, \ldots, 1/\sqrt{n})$ and $(c_1, \ldots, c_n)$.

(b) Let $(X_i, Y_i)$, $i = 1, \ldots, n$ be iid bivariate normally distributed with parameters $\mu_x, \mu_y, \sigma_x, \sigma_y$, and $\rho$. Show that the distribution of the sample correlation coefficient

$$r = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^n (X_i - \bar{X})^2 \sum_{i=1}^n (Y_i - \bar{Y})^2}}$$

does not depend on $\mu_x, \mu_y, \sigma_x$ or $\sigma_y$.

(c) Show that the conditional distribution of the sample correlation coefficient given $X_1, \ldots, X_n$ in (b) satisfies

$$\sqrt{n - 2} \frac{r}{\sqrt{1 - r^2}} \sim t_{n-2}$$

if $\rho = 0$ and $\sum_{i=1}^n (X_i - \bar{X})^2 > 0$. Hint: Use (a).

(d) Conclude from (c) that

$$\sqrt{n - 2} \frac{r}{\sqrt{1 - r^2}} \sim t_{n-2}$$
if $\rho = 0$.

2. For the two-sample problem in Lecture 34, fill in the details in the derivation of the form of the likelihood ratio test.

3. 4.1.3

4. 4.2.8

5. 4.3.1

6. 4.3.4