Dynamical Systems (550.391)
Take-Home Project I (Due: Thursday, October 13, 2005)

General Directions: This project is open book, open notes. That is, you may use the course text and any notes from Fall 2005 in completing this project. You are to work individually. If you have any questions about the project, please contact Prof. Castello.

Show all work and document any assumptions to receive full credit on a problem. All problems are to be done by hand unless otherwise stated.

1. **(20 pts)** Strogatz: Problem 2.3.4

2. **(50 pts)** Strogatz: Problem 3.7.6 (a)-(j)

3. **(15 pts)** Consider the linear system

$$\frac{dx}{dt} = hx - 4y; \qquad \frac{dy}{dt} = x + hy$$

where h is constant. Discuss the effect of the value of h on the type and stability of the critical point (0,0).

4. **(15 pts)** Consider the linear system

$$\frac{dx}{dt} = -x - hy; \qquad \frac{dy}{dt} = x - y$$

where h is constant. Discuss the effect of the value of h on the type and stability of the critical point (0,0).

5. **(20 pts)** This problem deals with the almost linear system

$$\frac{dx}{dt} = y + hx(x^2 + y^2) \qquad \frac{dy}{dt} = -x + hy(x^2 + y^2)$$

(a) What is the type and stability of the critical point (0,0) when $h = 0$?

(b) Suppose that $h \neq 0$. Let $r^2 = x^2 + y^2$. Show that $dr/dt = hr^3$.

(d) Suppose that $h = -1$. Integrate the differential equation in (b). What is $\lim_{t\to\infty} r(t)$? What can you say about the type and stability of the critical point (0,0) in this case?

(e) Suppose that $h = 1$. Integrate the differential equation in (b). What is $\lim_{t\to\infty} r(t)$? What can you say about the type and stability of the critical point (0,0) in this case?

6. **(30 pts)** Find all critical points for each of the given systems, and investigate the type and stability of each. Construct a phase portrait for each system using a computer system or graphing calculator.

(a)

$$\frac{dx}{dt} = y^2 - 1 \qquad \frac{dx}{dt} = x^3 - y$$

(b)

$$\frac{dx}{dt} = xy - 2 \qquad \frac{dx}{dt} = x - 2y$$