Dynamical Systems(550.391)
Homework 8 (Due Monday, December 05, 2005)

General Directions: You must show all work and document any assumptions to receive full credit on a problem. Feel free to use MATLAB or any other computer system to find the fixed points, eigenvalues, and, if necessary, eigenvectors for your analysis. (Refer to the Software Usage Guidelines on the course website to determine what computer output you need to submit.)

1. Strogatz: Problem 8.3.2

2. Plot a bifurcation diagram for the system

$$\dot{r} = r\left((r-1)^2 - \mu r\right) \qquad \dot{\theta} = 1.$$

   Give a possible reason as to why this type of bifurcation diagram should be known as a *fold bifurcation*.

   Comment: You'll have to convert back to Cartesian coordinates.

3. Plot a bifurcation diagram for the system

$$\dot{r} = r\left(\mu - 0.2r^6 + r^4 - r^2\right) \qquad \dot{\theta} = -1$$

   and indicate the regions where the system is multistable and/or possibly bistable.

   Comment: You'll have to convert back to Cartesian coordinates.

4. Find and classify all fixed points of the system

$$\begin{aligned} \dot{x} &= x + 2z \\ \dot{y} &= y - 3z \\ \dot{z} &= 2y + z. \end{aligned}$$

5. A three-dimensional Lotka-Volterra model is given by

$$\begin{aligned} \dot{x} &= x(1 - 2x + y - 5z) \\ \dot{y} &= y(1 - 5x - 2y - z) \\ \dot{z} &= z(1 + x - 3y - 2z). \end{aligned}$$

   (a) Identify the fixed points of the system that are in the first quadrant.

   (b) Show that $x + y + z = 0.5$ is a solution to this system.

   (c) Substitute $z = 0.5 - (x + y)$ into the system to derive a two dimensional model in x and y. Create a phase portrait in x, y-space and interpret what you observe in the context of a population model.

(d) Repeat part(c) but substitute for x and then y.

6. Select one of the following dynamical systems for study for your final project (content details forthcoming; the required articles for most of the topics are available on the course website)

   (a) Chua's Circuit (for background, see L.P. Shil'nikov, "Chua's circuit: Rigorous results and future problems," *International Journal of Bifurcation Chaos*, 4(1994), 489-519, although a different article may be required.)

   (b) Belousov-Zhabotinsky Chemical Reaction (see Sec 12.4 in text for an overview)

   (c) Rikitake's model of geomagnetic reversals (See Exercise 9.2.6)

   (d) Chen's System

   (e) SIQR model for childhood diseases

   (f) Infectious disease model

   (g) Forest Dynamics

   (h) Laser model

   (i) Modified Lorenz System

   (j) Rossler System (See Sec 12.3 for overview)