Stochastic Math Models (550.252)
Homework 5 (Due Thursday, October 20, 2011)

General Directions: You must show all work and document any assumptions to receive full credit. When formulating models, make sure to define your variables and label your objective function and constraints. Solve all linear systems using Excel. All other work should be done by hand unless otherwise stated.

1. In commuting to work I must first get on a bus near my house and then transfer to a second bus. If the waiting time in minutes at each stop is uniformly distributed betweeh $a = 0$ and $b = 5$ then it can be shown that my total waiting time Y has the pdf

$$f(y) = \begin{cases} \frac{1}{25}y & 0 \leq y < 5 \\ \frac{2}{5} - \frac{1}{25}y & 5 \leq y \leq 10 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Use Excel to create a graph of $f(y)$.

   (b) Verify that

$$\int_{-\infty}^{\infty} f(y)dy = 1$$

   (c) What is the probability that the total waiting time is at least 3 minutes?

   (d) What is the probability that the total waiting time is no more than 8 minutes?

   (e) What is the probability that the total waiting time is between 2 and 6 minutes?

2. Let X be a continuous random variable with cdf

$$F(x) = \begin{cases} 0 & x \leq 0 \\ \frac{x}{4}\left[1 + \ln \frac{4}{x}\right] & 0 \leq y \leq 4 \\ 1 & x > 4 \end{cases}$$

   (a) What is $Pr(X \leq 1)$

   (b) What is $Pr(2 < X \leq 3)$

   (c) What is the pdf of X?

3. There are two machines available for cutting corks intended for use in wine bottle. The first machine produces corks with diameters that are normally distributed with mean 3cm and standard deviation 0.1cm. The second machine produces corks with diameters that are normally distributed with mean 3.04cm and standard deviation 0.02cm. Acceptable corks have diameters between 2.9 and 3.1 cm. Which machine has a higher probability of producing an acceptable cork?

4. The random variable X is said to follow an Erlang distribution with parameters λ and n if

$$f(x;\lambda,n) = \begin{cases} \frac{\lambda(\lambda x)^{n-1}e^{-\lambda x}}{(n-1)!} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Here n is a positive integer. It can be shown that if the times between successive events are independent, each with an exponential distribution with mean $1/\lambda$, then the total time X that elapses before all of the next n events occurs has the given pdf.

(a) To find $E(X)$ you will have to use integration by parts. Do so twice. Based on what you come up with, argue why

$$E(X) = \frac{n}{\lambda}$$

(b) If the time in minutes between arrivals of successive customers is exponentially distributed with mean $\lambda = 0.5$, how much time can be expected to elapse before the tenth customer arrives?

5. Suppose a random variable X has an unknown continuous distribution. You do know that the minimum value is 10, the most likely value is 18 and the maximum possible value is 22.

(a) What is the pdf of X if you assume X follows a triangular distribution?

(b) What is the pdf of X if you assume X follows a Beta distribution?

Hint, estimate the mean and standard deviation using the formulas below

$$\mu = \frac{a+4m+b}{6} \qquad \sigma = \frac{b-a}{6}$$

Then solve for α and β using the formulas below

$$\mu = a + (b-a)\frac{\alpha}{\alpha+\beta} \qquad \sigma^2 = \frac{(b-a)^2\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$