Stochastic Math Models (550.252)
Homework 4 (Due Thursday, September 29, 2011)

General Directions: You must show all work and document any assumptions to receive full credit. When formulating models, make sure to define your variables and label your objective function and constraints. Solve all linear systems using Excel. All other work should be done by hand unless otherwise stated.

1. In a gambling game, a player bets on one of the numbers 1, 2, 3, 4, 5 or 6. Three dice are then rolled and if the number bet by the player appears k times ($k = 1, 2, 3$) the player wins $1.5k$ dollars. If the number bet does not appear on any of the dice, the player loses \$3. A game is considered fair to the player if the expected winnings are nonnegative. Is this game fair?

2. Suppose earthquakes occur in the western portion of the United States according to a Poisson distribution with mean rate of 2 earthquakes per week.

   (a) What is the probability of at least 3 earthquakes occurring within the next 2 weeks?

   (b) What is the probability distribution of the time, starting from now, until the next earthquake?

3. Calculating the variance of a random variable.

   (a) Show that for ANY random variable $E(X(X-1)) = E(X^2) - E(X)$.

   (b) How could you use the identity in part (a) to find $Var(X)$ for ANY random variable.

   (c) Now suppose X is a Poisson random variable with mean λ. Use the definition of $E(h(X))$ (as opposed to the shortcut formula in part (a)) find $E(X(X-1))$. (Hint: Let $y = x - 2$.)

   (d) Use your result from part (c) to show that the variance of a Poisson random variable with mean λ is also λ.

4. Suppose that the length of a phone call in minutes follows an exponential distribution with an average of 5 minutes per call. If someone arrives immediately ahead of you at a public telephone booth and you decide to wait until he is finished (instead of looking for another phone) what is the probability that you will have to wait

   (a) more than 6 minutes?

   (b) between 4 and 10 minutes?

5. A variation of the exponential distribution is the *Laplace* (or *double exponential*) distribution. Its density is given by

$$f(x) = \frac{1}{2}\theta e^{-\theta|x|} \quad -\infty < x < \infty$$

Find the cumulative distribution function for a Laplacian random variable. (Hint: you will need to consider two cases: $x < 0$ and $x > 0$.)

6. Suppose that a binary message (either 0 or 1) must be transmitted by cable from location A to location B. However, the data sent over the cable are subject to noise disturbance. To reduce the probability of error, the value of 2 is sent when the intended message is 1 and the value of -2 is sent when the intended message is 0.

Let x be the message sent from A, r the message received at B, and n the noise so that $r = x + n$. When the message is received at B it is decoded according to the following rule:

- If $r \geq 0.5$ then the message is 1.
- If $r < 0.5$ then the message is 0.

Suppose the noise is Laplacian with parameter $\lambda = 1$. Compute

(a) Pr(error | true message is 1)

(b) Pr(error | true message is 0)