

# Stochastic Math Models (550.252) Exam II

Friday, November 18, 2011

**General Directions:** This exam is closed book. You may use one 8.5x11 sheet of notes. To receive credit for a problem you must **SHOW ALL WORK** and write neatly and clearly. Please use the space provided. If you require extra space, please feel free to write on the back or attach extra sheets.

PRINT Name Solutions

<i>Problem</i>	<i>Points Earned</i>	<i>Points Possible</i>
1		20
2		20
3		15
4		25
5		10
6		10
Total		100

*I attest that I have completed this exam without unauthorized assistance from any person, materials, or device.*

Please SIGN and DATE in INK.

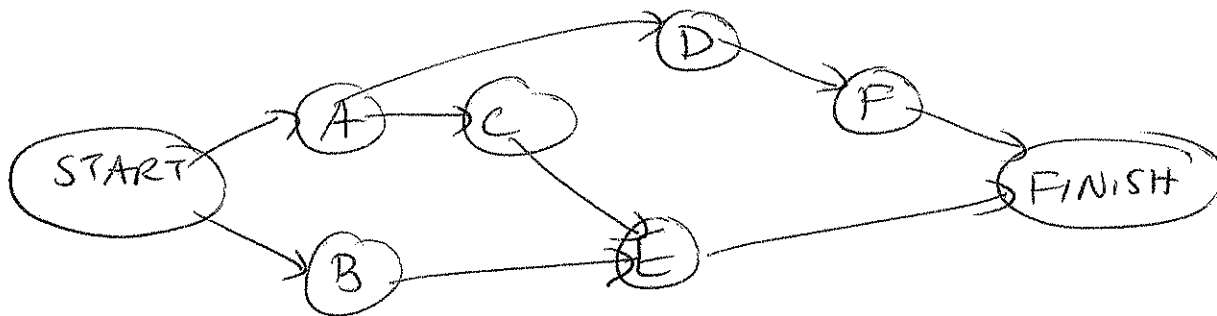
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1. Consider the following project information. (Times are given in weeks.)

Activity	Predecessor	<i>a</i> Optimistic	<i>m</i> Most Likely	<i>b</i> Pessimistic	Mean	Std Dev
A	-	1	4	7	4	1
B	-	1	3	5	3	$\frac{2}{3}$
C	A	1	2	9	3	$\frac{4}{3}$
D	A	2	3	10	4	$\frac{4}{3}$
E	B,C	1	2	3	2	$\frac{1}{3}$
F	D	2	5	8	5	1

Complete the table above.

$$\mu = \frac{a + 4m + b}{6} \quad \sigma = \frac{b - a}{6}$$



2. Using the data from Problem 1, fill in the table below.

Activity	ES	EF	LS	LF	Slack	
A	0	4	0	4	0	*
B	0	3	8	11	8	
C	4	7	8	11	4	
D	4	8	4	8	0	*
E	7	9	11	13	4	
F	8	13	8	13	0	*

critical path  $A \rightarrow D \rightarrow F$

3. Using the data in Problem 1, answer the following questions.

- (a) What is the mean and variance of the project's completion time?
- (b) Explain why we can estimate the standard deviation of the project's completion time as  $\sigma \approx 2$ .
- (c) Let  $T$  be the random variable denoting the project's completion time. Compute

$$\Pr(T \leq 16)$$

$$\begin{aligned} \text{(a)} \quad \mu_{\text{proj}} &= 13 \quad \sigma_{\text{proj}}^2 = \sigma_A^2 + \sigma_D^2 + \sigma_F^2 = 1 + \left(\frac{4}{3}\right)^2 + 1^2 \\ &= 1 + \frac{16}{9} + 1 = \frac{18+16}{9} = \frac{34}{9} \end{aligned}$$

$$\text{(b)} \quad \sigma_{\text{proj}} = \sqrt{\frac{34}{9}} \approx \sqrt{\frac{36}{9}} = \sqrt{4} = 2$$

$$\begin{aligned} \text{(c)} \quad \Pr(T \leq 16) &\approx \Pr\left(Z \leq \frac{16-13}{2}\right) \\ &= 0.5 + \Pr(0 \leq Z \leq 1.5) \\ &= 0.5 + .4332 = 0.9332 \end{aligned}$$

4. Consider the following payoff table (payoffs represent profits in million dollars which the decision maker wishes to maximize.)

	State 1	State 2	State 3	State 4	min
Alternative 1	3	2	1	5	1
Alternative 2	6	4	3	2	2 *
Alternative 3	10	7	2	-10	-10
Pr(State)	0.1	0.3	0.4	0.2	

- (a) If the decision maker is pessimistic (towards payoffs) what should his/her decision be? (Report ties, if any.)
- (b) If the decision maker uses the expected value rule what should his/her decision be? (Report ties, if any.)
- (c) What is the expected value of perfect information for this decision?

(a) pick maximin : alt 2

$$(b) EV(1) = \frac{1}{10}(3 + 3(2) + 4(1) + 2(5)) = \frac{23}{10}$$

$$EV(2) = \frac{1}{10}(6 + 3(4) + 4(3) + 2(2)) = \frac{34}{10} *$$

$$EV(3) = \frac{1}{10}(10 + 3(7) + 4(2) + 2(-10)) = \frac{19}{10}$$

pick max EV : alt 2

$$(c) EVPI = ERPI - EREV$$

$$ERPI = \frac{1}{10}(10 + 3(7) + 4(3) + 2(5)) = \frac{53}{10}$$

$$EREV = \frac{34}{10}$$

$$EVPI = \frac{19}{10}$$

5. You are about to take a final exam. Based on your prior experience, you have the following probability information:

$$\Pr(\text{Final Grade A}) = 0.2$$

$$\Pr(\text{Final Grade B}) = 0.4$$

$$\Pr(\text{Final Grade C}) = 0.3$$

$$\Pr(\text{Final Grade D}) = 0.1$$

Prior to taking the exam, you take a practice test. The test market is graded as *pass* or *fail*. Based on previous experience of this type, you believe the following conditional probabilities are correct:

$$P(\text{Pass practice exam} \mid \text{Final Grade A}) = 0.4$$

$$P(\text{Pass practice exam} \mid \text{Final Grade B}) = 0.9$$

$$P(\text{Pass practice exam} \mid \text{Final Grade C}) = 0.8$$

$$P(\text{Pass practice exam} \mid \text{Final Grade D}) = 0.2$$

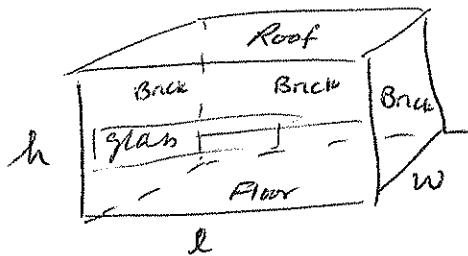
What are the posterior probabilities if you FAIL the practice exam?

State	Prior	Cond	Joint	Posterior
A	.2	.6	.12	$\frac{12}{30} = \Pr(A \mid \text{Fail})$
B	.4	.1	.04	$\frac{4}{30} = \Pr(B \mid \text{Fail})$
C	.3	.2	.06	$\frac{6}{30} = \Pr(C \mid \text{Fail})$
D	.1	.8	.08	$\frac{8}{30} = \Pr(D \mid \text{Fail})$
SUM			.3	

6. A departmental store plans to construct a one-story building in the shape of a rectangular prism. The building is required to have a floor area of at least 22,500 square feet and a height between 18 and 20 feet. The length of the building should be no more than twice the width of the building.

It is proposed to use brick walls on three sides and a glass wall on the fourth side. Glass costs twice as much per square foot as brick and the roof material cost three times as much as brick per square foot. Brick costs \$100 per square foot.

Formulate (DO NOT SOLVE) a nonlinear optimization model to find the dimensions (length, width, and height) of the building that minimize the cost of construction. (The cost of the floor/foundation is not relevant.)



$l = \text{length}$   
 $w = \text{width}$   
 $h = \text{height}$

$$\text{min } \underbrace{2(100)wh + 1(100)hl}_{\text{brick sides}} + \underbrace{1(200)lh}_{\text{glass}} + \underbrace{1(300)lw}_{\text{roof}}$$

$$\text{s.t. } 18 \leq h \leq 20 \quad (\text{height range})$$

$$lw \leq 22500 \quad (\text{floor area})$$

$$l \leq 2w \quad (\text{length-to-width-ratio})$$

$$l, w, h > 0 \quad (\text{positive dimensions})$$