APPENDIX 8.5

Derivation of the Planned Shortage Model

Figure A8.4 represents the inventory cycle for the planned shortage model. We see that the inventory cycle time, T, can be divided into two parts: T_1, representing the portion of the cycle during which inventory is available and T_2 representing the portion of time there is no inventory and customers must backorder.

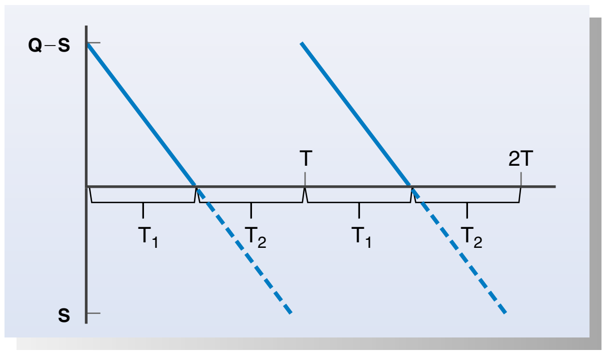

FIGURE A8.4

HOLDING COSTS

Annual holding costs are based on the average inventory level. Hence, one must derive an expression for this value. From Exhibit A8.4 note that each time a new order of Q units arrives, S units are on backorder and the maximum inventory level increases to Q − S. Then, at the end of time T_1 the inventory is depleted.

Hence, the average inventory level during time T_1 is $(Q - S)/2$.

During time T_2 the firm has no inventory, and average inventory is 0. To find the overall average inventory level during each cycle, the two averages are weighted by their respective times divided by the total cycle time. Therefore,

$$\text{Average Inventory Level} = \left(\frac{Q-S}{2}\right)\left(\frac{T_1}{T}\right) + 0\left(\frac{T_2}{T}\right) = \left(\frac{Q-S}{2}\right)\left(\frac{T_1}{T}\right)$$

Since Q units are demanded during time T and Q − S units are demanded during time T_1,

$$Q = DT \quad \text{or} \quad T = Q/D$$

and

$$Q - S = DT_1 \quad \text{or} \quad T_1 = \frac{Q-S}{D}$$

Substituting for T and T_1 gives:

$$\text{Average Inventory Level} = \frac{(Q-S)^2}{2Q}$$

Thus,

$$\begin{aligned}\text{Annual Holding Cost} &= (\text{Average Inventory Level}) * (\text{Annual Holding Cost per Unit}) \\ &= \frac{(Q-S)^2}{2Q} C_h\end{aligned}$$

ORDERING COSTS

$$\text{Annual Ordering Cost} = (\text{Number of Orders Per Year}) * (\text{Ordering Cost})$$
$$= \left(\frac{D}{Q}\right) C_o$$

BACKORDER COSTS

The annual backorder cost is dependent on the average backorder level and the number of backordered customers per year. To calculate the average backorder level, the same approach taken to determine the average inventory level is followed. During time T_1 there are no backorders; hence, the average backorder level is 0. During time T_2 the average backorder level is S/2. Hence,

$$\text{Average Backorder Level During a Cycle} = 0\left(\frac{T_1}{T}\right) + \frac{S}{2}\left(\frac{T_2}{T}\right) = \frac{S}{2}\left(\frac{T_2}{T}\right)$$

But since S units are demanded during time T_2, we have

$$S = DT_2 \quad \text{or} \quad T_2 = \frac{S}{D}$$

Substituting for T_2 and T gives:

$$\text{Average Backorder Level} = \frac{S^2}{2Q}$$

The number of backordered customers per year can be determined by recognizing that during each inventory cycle there are S backorders and there are D/Q inventory cycles per year. Hence,

$$\text{Number of Backorders per Year} = S\left(\frac{D}{Q}\right)$$

Thus,

Total Annual Backorder Cost =
(Average Backorder Level) (Annual Backorder Cost per Unit)
\+ (Number of Backorders During the Year) (Administrative Backorder Cost Per Unit)

$$= \frac{S^2}{2Q} C_s + S\left(\frac{D}{Q}\right) C_b$$

TOTAL VARIABLE COSTS

Combining these terms gives us the following formula for TV(Q, S):

$$TV(Q, S) = \frac{(Q - S)^2 C_h}{2Q} + \frac{D(C_o + SC_b)}{Q} + \frac{S^2 C_s}{2Q} \qquad \text{(A8.16)}$$

Taking partial derivatives of TV(Q, S) with respect to Q and S and solving the resulting equations yields the results for Q* and S* given in Equations 8.11 and 8.12 of the text.