solution, the problem must be re-solved. Deleting a variable that was nonzero in the original optimal solution will result in a worse objective function value or one that is at best no better than the original objective function value.

ADDITION OF A VARIABLE

When a variable is added, in most cases, the problem must be re-solved. There is, however, a **net marginal profit** procedure that can determine whether the addition of the new variable will have any effect on the optimal solution. Net marginal profit is the difference between the objective function coefficient and the total marginal cost of the resources (calculated using the current values of the shadow prices).

To illustrate, suppose that a new product, Big Squirts, requiring three pounds of plastic and five minutes of production time, can be produced, yielding a profit of $10 per dozen. The new model is:

$$
\begin{array}{lrll}
\text{MAXIMIZE} & 8X_1 + 5X_2 + 10X_3 & & \\
\text{ST} & 2X_1 + X_2 + 3X_3 & \leq 1000 & \text{(Plastic)} \\
 & 3X_1 + 4X_2 + 5X_3 & \leq 2400 & \text{(Production time)} \\
 & X_1 + X_2 + X_3 & \leq 700 & \text{(Total units)} \\
 & X_1 - X_2 + & \leq 350 & \text{(Space Ray/Zapper mix)} \\
 & X_1, X_2, X_3 & \geq 0 &
\end{array}
$$

The shadow prices for the constraints turn out to be $3.40, $0.40, $0, and $0, respectively. Thus, the net marginal profit for the production of a dozen Big Squirts is:

$$\$10 - ((\$3.40)(3) + (\$0.40)(5) + (\$0)(1) + (\$0)(0)) = -\$2.20$$

Hence, it would not be profitable to produce Big Squirts, and the current solution of producing 320 dozen Space Rays and 360 dozen Zappers remains optimal. If the profit per dozen Big Squirts had been $15, however, the net marginal profit would have been $2.80. This would indicate that there is a new optimal solution which includes the production of Big Squirts, yielding a higher optimal profit.

CHANGES IN THE LEFT-HAND SIDE COEFFICIENTS

When a left-hand side coefficient is changed, the entire feasible region is reshaped. If the change is made to a coefficient in a nonbinding constraint, the first step is to ascertain whether the current optimal solution satisfies the modified constraint. If it does, it remains the optimal solution to the revised model; if it does not, or if the change is made in a coefficient of a binding constraint, both the optimal solution and the shadow prices change in ways that are more complex to calculate than changes resulting from modifications to objective function coefficients or right-hand side values. In this case, the model must be re-solved.

2.5 Using Excel Solver to Find an Optimal Solution and Analyze Results

In this section, we illustrate the use of Excel's Solver to determine an optimal solution for the Galaxy Industries model and discuss the information generated by its sensitivity reports. The step-by-step procedure outlined here is very valuable, for it can be used to solve any linear model with any number of decision variables.

Solver is an option found in the Tools menu. If you do not see Solver listed, you must check the Solver Add-In box of the Add-Ins option under the Tools menu. If you do not see this option listed, you may have to re-install part or all of Excel using the disks that include the Solver option.

To use Solver, designate cells to contain:

- The values of the decision variables (known as **changing** or **adjustable cells**)
- The value of the objective function (known as the **target cell**)
- The total value of the left-hand side of the constraints

The cells for the objective function value and the left hand side values contain formulas that can be written as the sum of several terms. While the complete formulas can be entered explicitly, using Excel's SUM and SUMPRODUCT functions usually simplifies the input.

We shall use the spreadsheet developed in Figure 2.13 to solve the Galaxy Industries model. Although color coding and boxing cells can have an impact on the effect of the presentation of the spreadsheet (we discuss this in more detail in Chapter 3), here we concentrate on the basics of Solver.

Galaxy.xls

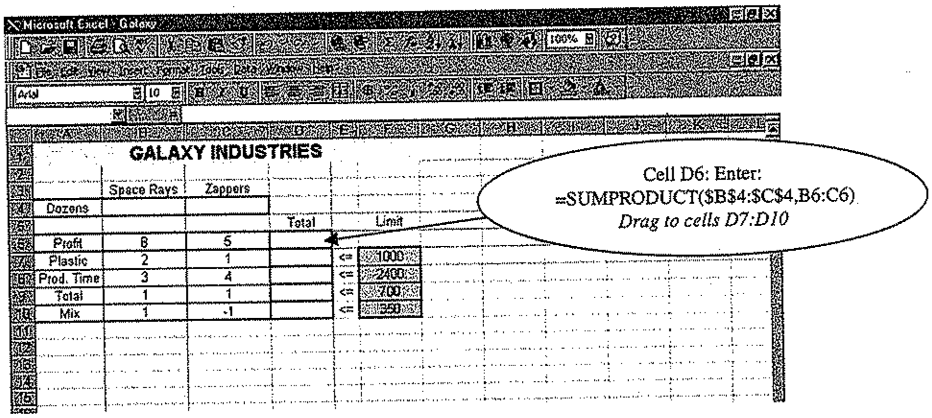


FIGURE 2.13
Excel Spreadsheet for Galaxy Industries

Begin by entering the data for the model as shown. Designate cells B4 and C4 as the adjustable cells that will contain the values of the decision variables for Space Rays and Zappers. Cell D6 will be "programmed" to contain the value of the objective function, and cells D7 to D10 (written as D7:D10) will be programmed to contain the left-hand side of the constraints.

ENTERING FORMULAS FOR THE OBJECTIVE FUNCTION AND LEFT-HAND SIDE VALUES

The formula for the total profit in cell D6 could be written as B4*B6 + C4*C6. This is fine when there are only two variables. But when there are many variables, this is more easily done using the SUMPRODUCT function of Excel. This function has the form =SUMPRODUCT(*array1*, *array2*). Here array 1 and array 2 are rows or columns of equal length.[4] The arrays can be entered by highlighting

[4] If one of the arrays is a row and the other is a column, the equivalent formula is =MMULT(array1, array2), where array 1 is the *row* and array 2 is the *column*.

the appropriate cells (cells B4 and C4 and cells B6 and C6 in this case) using the left mouse key. Thus, as shown in Figure 2.13, for cell D6, we enter:

=SUMPRODUCT(B4:C4,B6:C6)

Cells D7, D8, D9, and D10 will contain the total on the left side for the plastic production time, total production, and mix constraints, respectively, resulting from a given solution. Each has a formula similar to that in cell D6 with the same first array (B4:C4) and a second array that is relative to the row. To easily enter these formulas, return to cell D6. Highlight only the first array (B4:C4) of the formula in the formula bar and press the F4 function key at the top of the keyboard. This makes these cell references absolute by inserting $ signs. After pressing Enter, the formula in cell D6 is now:

=SUMPRODUCT($B$4:$C$4,B6:C6)

When we drag this formula to cells D7, D8, D9, and D10, the resulting respective formulas in those cells give the total left-hand side for each of the constraints. The formulas in these cells are shown in Table 2.6.[5]

TABLE 2.6 Cell Formulas in Figure 2.13

Cell	Quantity	Formula	Excel Formula
D6	Total Weekly Profit	$8X_1 + 5X_2$	=SUMPRODUCT($B$4:$C$4,B6:C6)
D7	Total Plastic Used Weekly	$2X_1 + 1X_2$	=SUMPRODUCT($B$4:$C$4,B7:C7)
D8	Total Production Minutes Used Weekly	$3X_1 + 4X_2$	=SUMPRODUCT($B$4:$C$4,B8:C8)
D9	Total Weekly Production	$1X_1 + 1X_2$	=SUMPRODUCT($B$4:$C$4,B9:C9)
D10	Amount Space Ray Production Exceeds Zapper Production Each Week	$1X_1 - 1X_2$	=SUMPRODUCT($B$4:$C$4,B10:C10)

We now call Solver from the Tools menu. This gives the dialogue box shown in Figure 2.14.

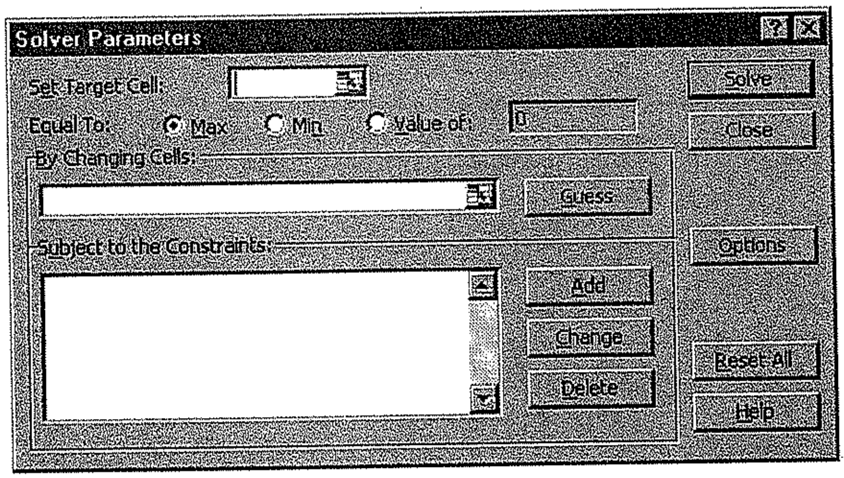


FIGURE 2.14
Solver Dialogue Box

[5] Alternatively, we could have *defined* the word **Dozens** to be cells B4:C4 by highlighting these cells and then going to **Insert** menu and selecting **Name**, then **Define** and click OK. The formula in cell D6 would now be programmed as =SUMPRODUCT(Dozens,B6:C6) which would be dragged to cells D7, D8, D9, and D10.

FILLING IN THE SOLVER DIALOGUE BOX

Step 1: Set Target Cell

The target cell is the cell containing the value of the objective function. This cell must have a formula in it. For this model, this is cell D6 since it will contain the total weekly profit.

> With the cursor in the Set Target Cell box: Click on Cell D6.

Step 2: Equal To

This tells Solver whether you want to find a solution that maximizes or minimizes the value of the objective function or find a solution that gives a particular value for the objective function. In linear programming we are always seeking to maximize or minimize the objective function value. For this model, we wish to maximize cell D6.

> Leave the button for Max highlighted.

Step 3: By Changing Cells

Changing cells are the cells that contain the decision variables. Solver will return values to these cells that optimize the objective function subject to the constraints entered below.[6] For this model the decision variables are in cells B4 and C4.

> With the cursor in the By Changing Cells box: Highlight Cells B4 and C4.

Step 4: Subject to the Constraints

To enter constraints we click the Add button. This brings up the Add Constraint dialogue box shown in Figure 2.15.

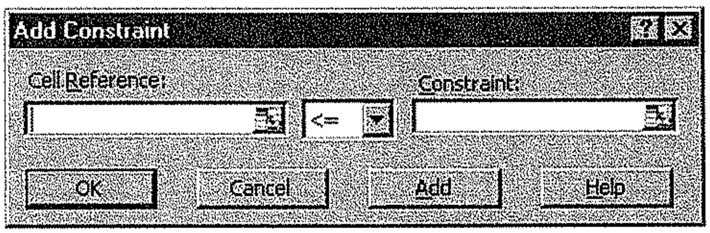


FIGURE 2.15
The Add Constraint Dialogue Box

Note that the default direction for a constraint is "≤," but this can be changed by clicking on the arrow of the drop down box. Options include "≤," "=," "≥," "int," and "bin." The last two options allow us to restrict a Cell Reference to be integer-valued or binary (0 or 1), respectively.

In the Cell Reference box, designate the cell containing the value for the left side of the constraint. In the Constraint box, input a number, a formula, or a cell containing the right side of the constraint. Several constraints can be entered at one time as long as they have the same direction. For our model, all the constraints are "≤," and thus we input them all at once.

[6] Note that if formulas are entered in any of the cells designated as Changing Cells, Solver will simply ignore these formulas.

With the cursor in the Cell Reference box: Highlight cells D7 through D10. Leave the direction as "≤." With the cursor in the Constraint box: Highlight cells F7 through F10.

If more constraints were to be added, we would click **Add** and follow the same procedure. When we are done entering constraints in the Add Constraint dialogue box and click OK.

Step 5: Options

Clicking Options brings up the dialogue box shown in Figure 2.16.

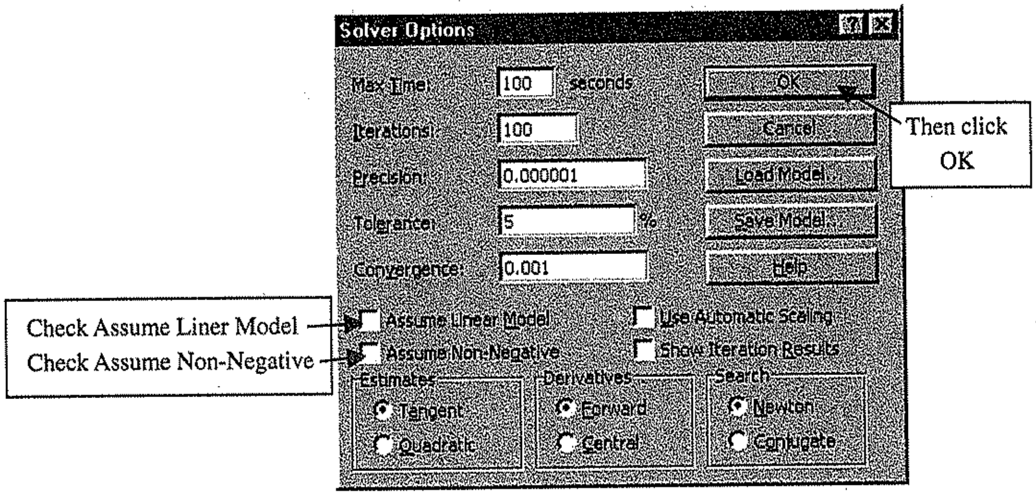


FIGURE 2.16 The Solver Options Dialogue Box

This dialogue box allows us to reset several parameters of a technical nature that are beyond the scope of our discussion here. However, it is important that we designate that the variables are restricted to be "≥0" (Assume Non-Negative) and that the problem be solved specifically as a linear program rather than a general mathematical programming model (Assume Linear). Doing these two things allows relevant "what-if" sensitivity analyses to be generated. Check these boxes and click OK.

Step 6: Solve

Figure 2.17 shows the completed Solver dialogue box for the Galaxy Industries model.

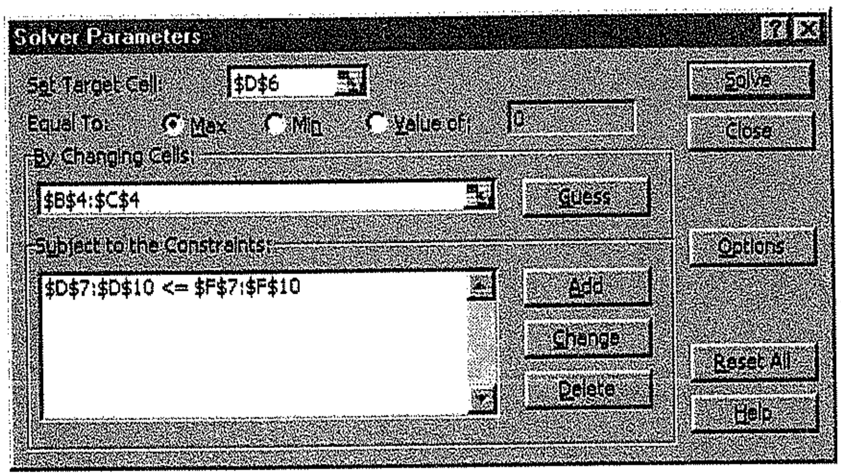


FIGURE 2.17
Completed Solver Dialogue Box for Galaxy Industries

To solve for the optimal solution, click Solve.

The values in the spreadsheet are changed to reflect the optimal solution, the optimal value of the objective function, and the total left-hand side values in their respective cells.

Step 7: Reports

On top of the spreadsheet is the Solver Results dialogue box shown in Figure 2.18.

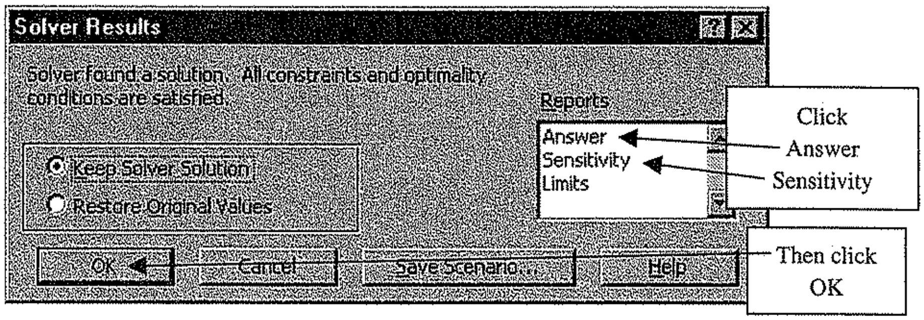


FIGURE 2.18
Solver Results Dialogue Box

This indicates that an optimal solution was found, it also asks which reports we wish to have generated. The two reports that are of interest to us are the Answer Report and the Sensitivity Report. Highlight them and click OK.

We are finally done! Now let us analyze the results.

Analyzing the Excel Spreadsheet

Figure 2.19 shows the optimal Excel spreadsheet for the Galaxy Industries model.

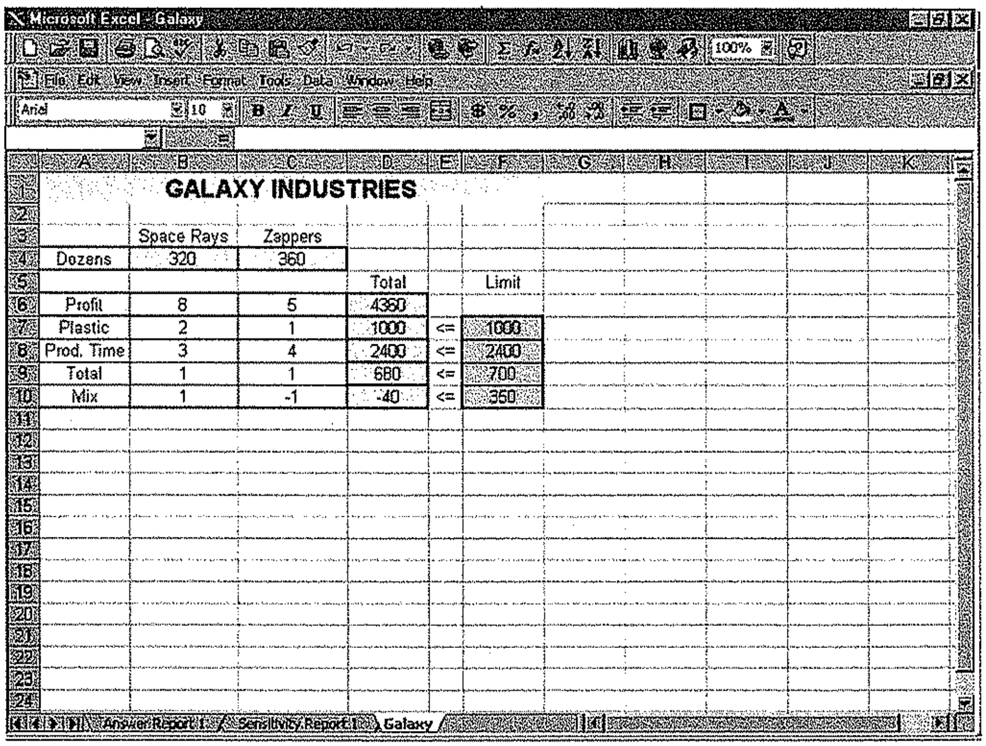


FIGURE 2.19 Optimal Excel Spreadsheet for Galaxy Industries

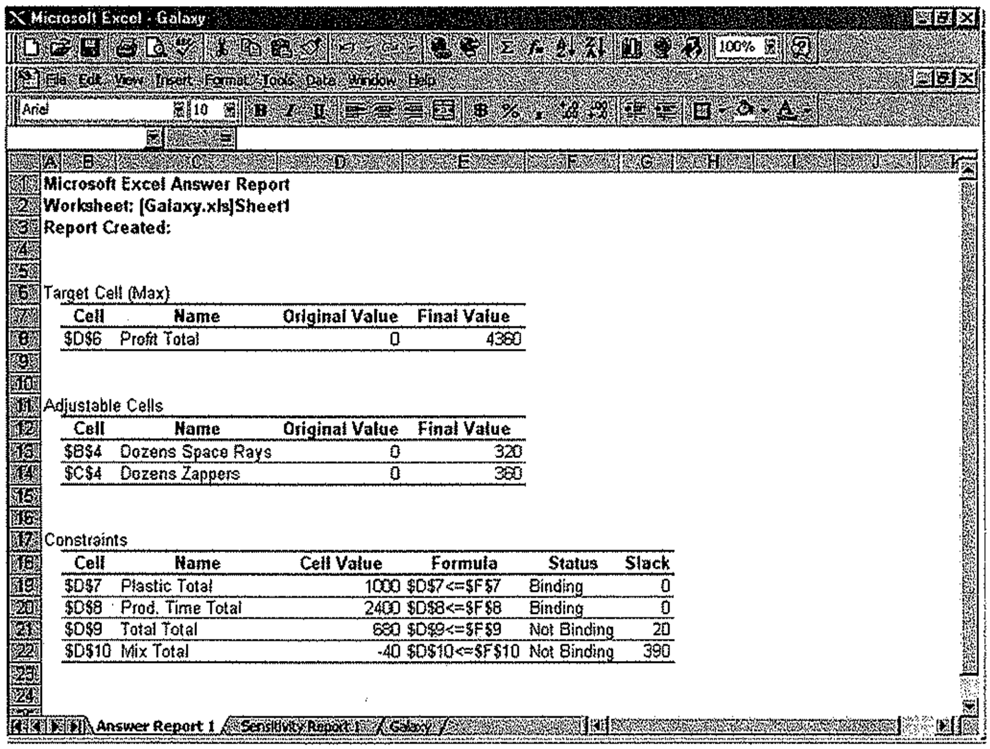
From this spreadsheet we can see that the optimal solution for this model is:

- Produce 320 dozen Space Rays (cell B4) and 360 dozen Zappers (cell C4) weekly
- Total weekly profit = \$4360 (cell D6)

From column D, we see that this solution uses all 1000 pounds of plastic (cell D7) and all 2400 production minutes (cell D8). Since these two constraints hold as *equalities* at the optimal solution, they are said to have *no slack* and they are the *binding constraints*. The left side of the total production constraint is 680 (cell D9), and it indicates that there is a *slack* of 20 for the total production constraint (calculated by subtracting the left-side value (680) from right-side limit for the constraint (700)). Similarly, the slack for the mix constraint is $350 - (-40) = 390$.

The Answer Report

The above information is summarized in the Answer Report. Clicking on the Answer Report tab at the bottom of the spreadsheet gives the window shown in Figure 2.20.



Microsoft Excel Answer Report
Worksheet: [Galaxy.xls]Sheet1
Report Created:

Target Cell (Max)

Cell	Name	Original Value	Final Value
\$D\$6	Profit Total	0	4360

Adjustable Cells

Cell	Name	Original Value	Final Value
\$B\$4	Dozens Space Rays	0	320
\$C\$4	Dozens Zappers	0	360

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$D\$7	Plastic Total	1000	\$D\$7<=\$F\$7	Binding	0
\$D\$8	Prod. Time Total	2400	\$D\$8<=\$F\$8	Binding	0
\$D\$9	Total Total	680	\$D\$9<=\$F\$9	Not Binding	20
\$D\$10	Mix Total	-40	\$D\$10<=\$F\$10	Not Binding	390

FIGURE 2.20 Galaxy Industries Answer Report

The Answer Report is divided into three sections: the Target Cell section, the Adjustable Cells section, and the Constraints section. What appears in the **Name** column of each of these sections is a combination of the last nonnumeric cell to the left and the last nonnumeric cell above the corresponding cell entry.

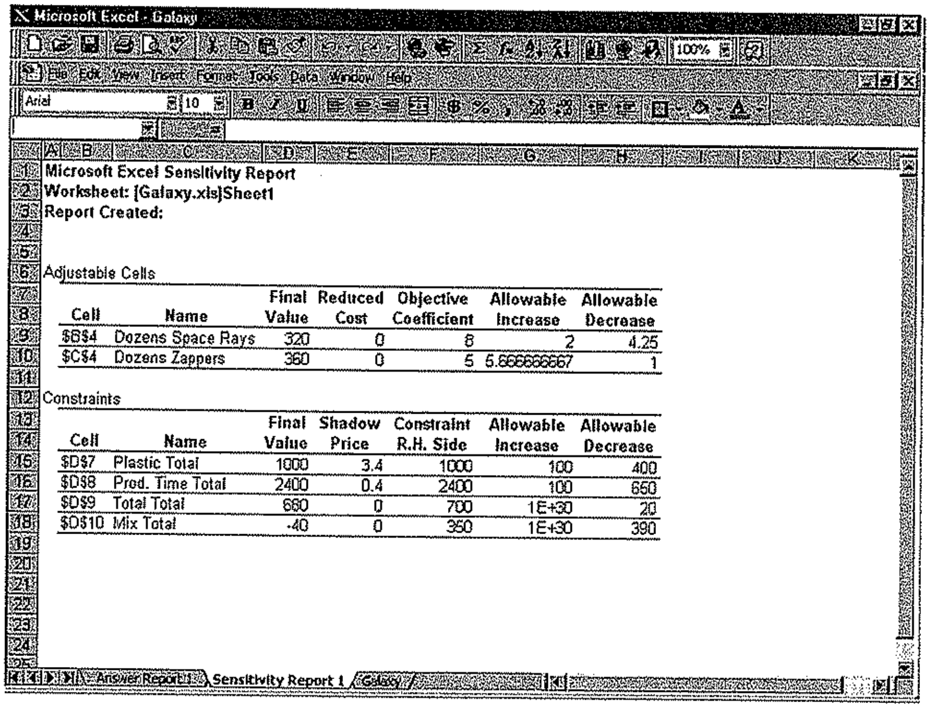
In the **Target Cell** Section, the optimal value of the objective function is given in the column labeled **Final Value**. Similarly, the optimal values for the decision variables are found in the **Final Value** column of the **Adjustable Cells** section.

In the **Constraints** section, the **Cell Value** column gives the total values of the left side of the constraints (i.e., the values in cells D7, D8, D9, and D10). The information entered in the Constraint Dialogue Box of Solver is given in the **Formula** column. The **Slack** column shows the amount of slack for each constraint.

Note that if the slack is 0, the word "Binding" is printed in the **Status** column; "Not Binding" is printed when the slack is positive.

The Sensitivity Report

The Sensitivity Report shown in Figure 2.21 contains the relevant information concerning the effects of changes to either an objective function coefficient or a right-hand side value as discussed in Section 2.4. The Adjustable Cells section includes the reduced costs and ranges of optimality for objective function coefficients (expressed in terms of Allowable Increases and Allowable Decreases). The Constraints section details the shadow prices and ranges of feasibility for right-hand side values (again expressed in terms of Allowable Increases and Allowable Decreases). This report can be thought of as the linear programming equivalent of a *marginal analysis* in economics, as the results deal with the effects of making *one and only one parameter value change* to the model.



Microsoft Excel Sensitivity Report
Worksheet: [Galaxy.xls]Sheet1
Report Created:

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$4	Dozens Space Rays	320	0	8	2	4.25
$C$4	Dozens Zappers	360	0	5	5.666666667	1

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$7	Plastic Total	1000	3.4	1000	100	400
$D$8	Prod. Time Total	2400	0.4	2400	100	650
$D$9	Total Total	680	0	700	1E+30	20
$D$10	Mix Total	-40	0	350	1E+30	390

FIGURE 2.21
Galaxy Industries Sensitivity Report

Note that in Figure 2.21 there is a value of "1E+30" for the Allowable Increase for the total production and mix constraints. "1E+30" is Excel's way of saying "infinity." That is, the range of feasibility for the total production is from $700 - 20 = 680$ to infinity, and the range of feasibility for the mix constraint is from $350 - 390 = -40$ to infinity.

2.6 Using Computer Output to Generate a Management Report

Given the computer solution to the problem faced by Galaxy Industries, the following report can be prepared for Hal Barnes, Galaxy's production manager. This report compares the results of the recommended policy to those of the current policy at Galaxy Industries. It not only summarizes the results but also details the distribution of the resources as well as sensitivity issues that might be of interest to management. The statements made about the shadow price for plastic and production hours outside their ranges of feasibility are based on completely re-solving the problem.