Supplement CD6

Production Based Inventory Models

Material Requirements Planning

Typically, a manufacturer produces many different finished goods. For example, a television set manufacturer may produce several different models, with variations in screen size and options such as remote control or stereo. Many of these finished goods share the same components (e.g., speakers, picture tubes). Further complicating the process is the fact that a finished product can consist of components that need assembly themselves. Thus, several layers or hierarchies might be involved in the assembly of the finished good, resulting in dependence among component demands.

Material requirements planning (MRP) is a computer-based technique used for controlling inventory in such cases. While the concepts of MRP illustrated in this supplement are developed in terms of keeping manual records rather than using a computer approach, it is important to realize that real-world applications of MRP are nearly always implemented using a computer.

MRP INPUTS

The key inputs into the MRP process are: (1) the master production schedule, (2) the bill of materials, and (3) the inventory records file.

The *master production schedule* is a forecast of the finished goods demand over a particular *planning horizon*, consisting of several weeks or even months of production. The *bill of materials* details how the finished good is to be assembled. The *inventory records file* contains information about the inventory status of each component for each period of time, as well as the component vendors, required lead time for delivery, and specified lot size. Included in the inventory information are the quantity on hand and on order for each component. Table CD6.1 summarizes the key elements of the MRP system.

Table CD6.1 Key Elements of an MRP System

Master Production Schedule	Bill or Materials	Inventory Records File
A forecast of demands for the finished good over a particular planning horizon	A detailed listing of how the finished good is assembled	Information on the inventory status of each component

An MRP system "translates" finished good demands into demands for their individual components, allowing the firm to properly manage its inventory records file. Although an MRP system looks at finished good demands over a given planning horizon, decisions that need to be made in the current period are of utmost importance to the manager. Because of component lead times, however, decisions made regarding components in the current period must be based on future demand projections for finished goods. Therefore, in implementing an MRP system, it is essential to extend the finished good planning horizon sufficiently far into the future to adequately account for such lead times.

To incorporate possible changes in demand forecasts, most firms use *rolling updates* of their master production schedule. That is, whenever changes are made to the master production schedule forecasts, the MRP reevaluates the inventory policy to determine the impact of such changes on the current period's inventory policy.

To illustrate the concepts of an MRP system, consider the manufacturing decisions faced by Little Trykes Toys, Inc.

LITTLE TRYKES TOYS, INC.

Little Trykes Toys, Inc. (LTT) manufactures children's tricycles and wagons. Figure CD6.1 shows the components required to make these two product. The components required for final assembly of a tricycle are a frame, seat, front wheel assembly, and small wheel set for the rear wheels. A front wheel assembly consists of a handle bar, a large wheel, and a pedal. The small wheel set consists of an axle and two small wheels. Each small wheel is purchased fully assembled, but the company produces the large wheels using a rim and a tire. The wagon assembly consists of a handle, wagon body, and two small wheel sets. Both the tricycles and wagons use the same small wheel sets.

LTT has received an order for delivery of 1000 tricycles 10 weeks from now and would like to schedule production and ordering of the subcomponents to meet this demand.

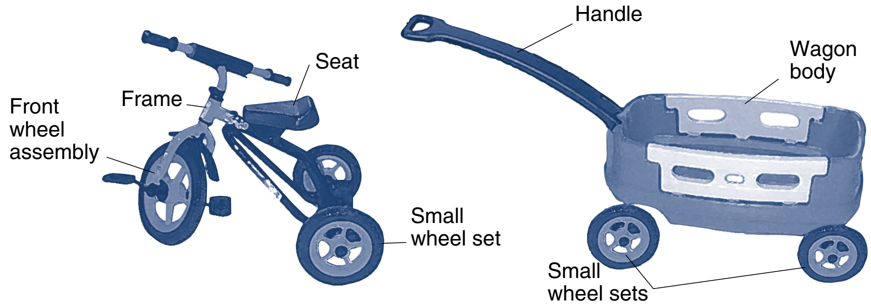


FIGURE CD6.1
Products Manufactured by Little Trykes Toys, Inc., and Their Level 1 Components

SOLUTION

PRODUCT TREES

One way to represent the bill of materials pictorially is by a *product tree*. Figures CD6.2 and CD6.3 give the product trees for the tricycle and wagon produced by LTT, respectively. The "root" of the product tree corresponds to the finished good and is placed at the top of the diagram; it is known as hierarchical level 0. Immediately below the root are the components or subassemblies required to produce the finished good. For the tricycle, (Figure CD6.2) this consists of the frame, seat, front wheel assembly, and small wheel set. These items correspond to hierarchical level 1.

Similarly, below each level 1 subassembly are the hierarchical level 2 components for the level 1 subassemblies. For example, below the tricycle front wheel assembly in Figure CD6.2 are the handle bars, pedals, and large wheels. Below the small wheel set assembly are the axles and small wheels. Because each small wheel set requires two small wheels, the number 2 is placed in parentheses within the box representing small wheels. The process continues in this fashion until all the components have been delineated.

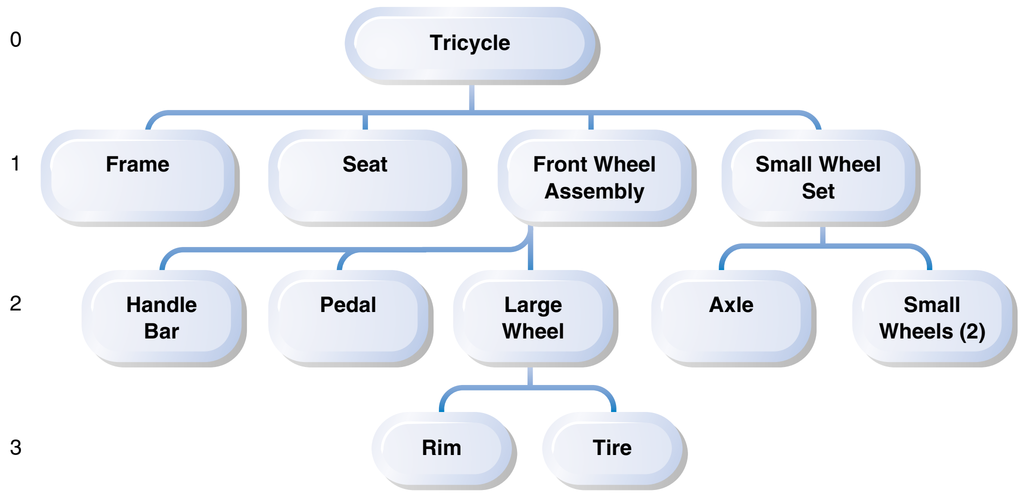


FIGURE CD6.2 LTT Product Tree for Tricycles

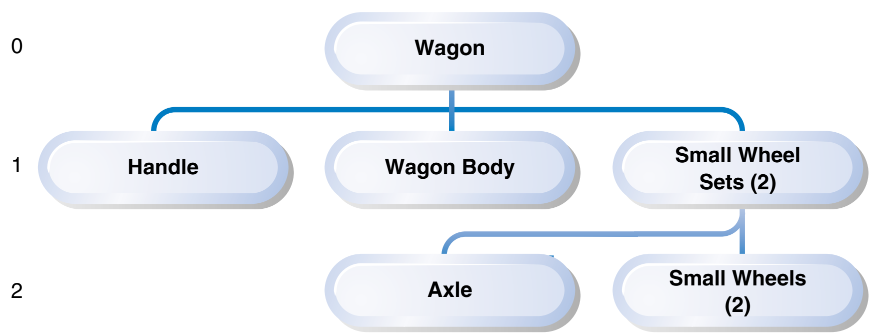


FIGURE CD6.3 LTT Product Tree for Wagons

Since the small wheel sets that Little Trykes uses for the wagon are the same as those used for the rear wheels of the tricycle, Little Trykes must keep track of a total of 15 different components, subassemblies, or finished goods. The lead times for each of these items have been determined and are summarized in Table CD6.2.

TABLE CD6.2 Lead Times for Tricycle and Wagon Components

Item	Lead Time
Tricycles	2 weeks
Frames	3 weeks
Seats	4 weeks
Front Wheel Assemblies	1 week
Handle Bars	2 weeks
Pedals	3 weeks
Large Wheels	1 week
Rims	2 weeks
Tires	1 week
Small Wheel Sets	2 weeks
Axles	2 weeks
Small Wheels	3 weeks
Wagons	1 week
Handles	5 weeks
Wagon Bodies	4 weeks

Because the lead time for assembling the tricycles is two weeks, LTT must begin tricycle assembly at the start of week 8 (= 10 − 2). The tricycles cannot be produced unless the frames, seats, front wheel assemblies, and small wheel sets are available. Each of these level 1 components must therefore be available by the start of week 8. Continuing this analysis, because frames have a lead time of three weeks, in order for them to be available by week 8, the firm must order them by week 5 (=8 − 3).

Modifying the product tree to include the intended delivery date and component lead times provides an easy way to determine the order date for each component. This is done by placing the component's lead time (in weeks) in square brackets ([ ]) inside each component's box (see Figure CD6.4). Then, beginning at the top of the tree with the intended *delivery date* (week 10 in this case), the *order date* for the finished good can be found by subtracting the lead time value from this intended delivery date.

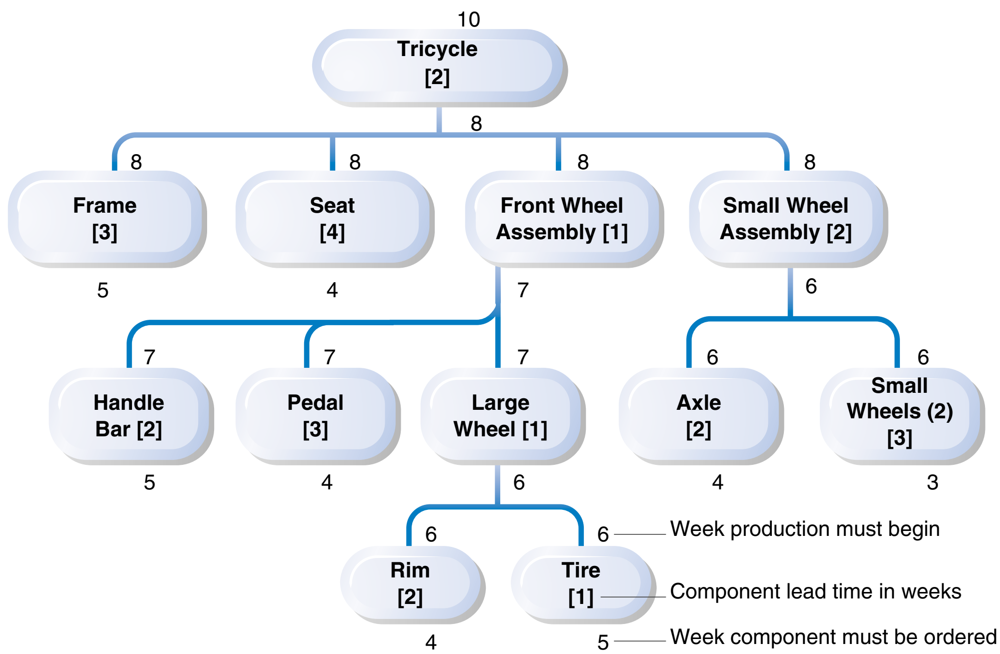


FIGURE CD6.4 Tricycle Component Lead Times

This procedure is repeated for subsequent hierarchical levels, with the intended delivery date for each component being the same as the ordering date of the component above it in the hierarchy. The order date for each component can then be found by subtracting its lead time value (found in that component's box) from the component's intended delivery date. The order dates necessary to support production in week 10, therefore, are as given in Table CD6.3.

Thus, if LTT wishes to produce tricycles beginning in week 10, it must order small wheels at the beginning of week 3, seven weeks prior to the intended completion date. Any planning cycle for a given week's tricycle production should include at least seven weeks if LTT is to account for all components properly.

TABLE CD6.3 Order Dates for Tricycle Components

Item	Order Date
Tricycles	week 10 − 2 = 8
Frames	week 8 − 3 = 5
Seats	week 8 − 4 = 4
Front Wheel Assemblies	week 8 − 1 = 7
Handle Bars	week 7 − 2 = 5
Pedals	week 7 − 3 = 4
Large Wheels	week 7 − 1 = 6
Rims	week 6 − 2 = 4
Tires	week 6 − 1 = 5
Small Wheel Sets	week 8 − 2 = 6
Axles	week 6 − 2 = 4
Small Wheels	week 6 − 3 = 3

NET INVENTORY REQUIREMENTS PER ORDER

If LTT does not have any inventory in stock for any of the components, it is fairly easy to determine how many of each component should be ordered. For example, if the firm wishes to produce 1000 tricycles, since each tricycle needs one seat, one frame, one front wheel assembly, and one rear wheel set, LTT will need a *gross inventory requirement* of 1000 units of each of these components.

Suppose, however, that LTT has an *available inventory* of 200 front wheel assemblies and 400 small wheel sets. As a result, it only needs to order 1000 − 200 = 800 front wheel assemblies and 1000 − 400 = 600 small wheel sets. These are the *net inventory requirements* for the hierarchical level 1 assemblies.

The net component requirements at each hierarchical level, can be found using the following relationship:

$$\begin{pmatrix}\text{Net Inventory}\\ \text{Requirements}\end{pmatrix} = \begin{pmatrix}\text{Gross Inventory}\\ \text{Requirements}\end{pmatrix} - \begin{pmatrix}\text{Available}\\ \text{Inventory}\end{pmatrix}$$

Continuing with hierarchical level 2, the 800 front wheel assemblies require 800 handle bars, 800 pedals, and 800 large wheels. The 600 small wheel sets require 600 axles but, since each small wheel set involves two wheels, a total of 2(600) = 1200 wheels are needed. Similarly, at hierarchical level 3, a total of 800 large tires and 800 rims are required for the 800 large wheels.

The product tree can be modified to provide an easy way to determine the net requirements for each component by including the current inventory level for each component in brackets ([ ]) inside each component box and the desired production level (1000 units) at the top of the tree. The desired production level corresponds to the gross component requirements for the finished good.

The gross requirements for a component are found by multiplying the net component requirements for the component at the hierarchical level directly above it by the number of units of the component going into the product. The component's net requirements are found by subtracting its current inventory from its gross requirements (a negative value corresponds to a net requirement of 0). For example, suppose LTT has the inventory in stock shown in Table CD6.4 to support the production of 1000 tricycles in week 10. The net requirements of each component are calculated using the product tree as depicted in Figure CD6.5.

TABLE CD6.4 Available Tricycle Component Inventory

Item	Inventory Level
Tricycles	200 units
Frames	100 units
Seats	400 units
Front Wheel Assemblies	none
Handle Bars	150 units
Pedals	300 units
Large Wheels	250 units
Rims	600 units
Tires	350 units
Small Wheel Sets	300 units
Axles	250 units
Small Wheels	400 units

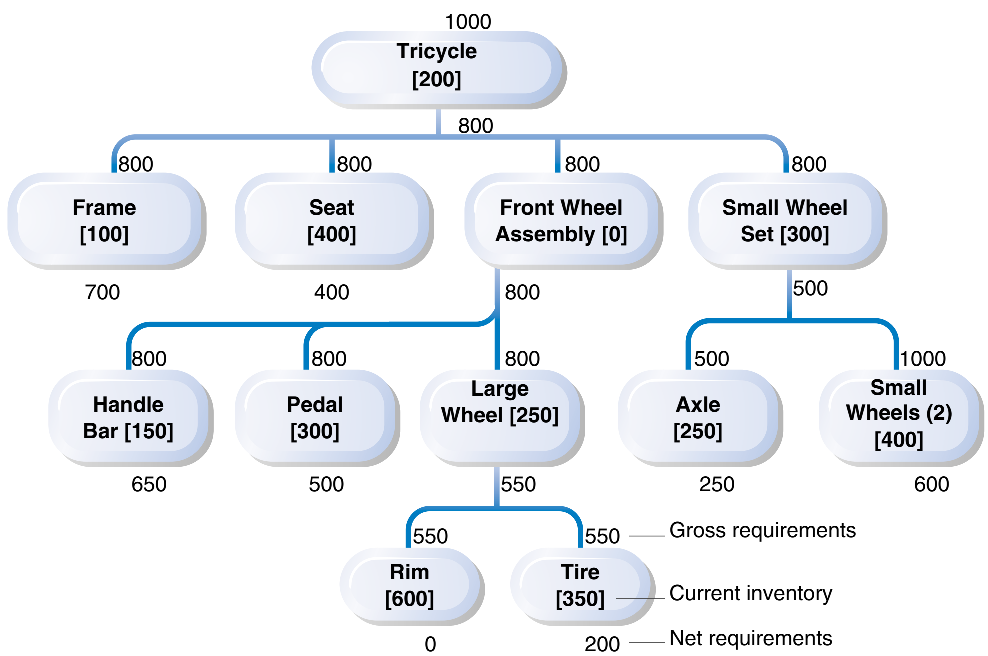


FIGURE CD6.5 Tricycle Net Requirements

To illustrate just one branch of the tree, note that, to support the production of 1000 tricycles, since 200 are already in stock, 800 must be produced. Thus, 800 small wheel sets are required. Since 300 small wheel sets are currently in inventory, 500 small wheel sets are needed. Each small wheel set requires an axle, and, since 250 axles are in inventory, 250 additional axles are required. Each small wheel set also requires two small wheels. Thus, the gross requirements for small wheels are 2(500) = 1000. Subtracting the available inventory of 400 small wheels from the gross requirements of 1000 yields a net requirement of 600 small wheels. Given the preceding product tree, Table CD6.5 summarizes the net component requirements.

TABLE CD6.5 Net Component Requirements

Item	Net Component Requirements
Tricycles	1000 − 200 = 800 units
Frames	800 − 100 = 700 units
Seats	800 − 400 = 400 units
Front Wheel Assemblies	800 − 0 = 800 units
Handle Bars	800 − 150 = 650 units
Pedals	800 − 300 = 500 units
Large Wheels	800 − 250 = 550 units
Rims	550 − 600 = none
Tires	550 − 350 = 200 units
Small Wheel Sets	800 − 300 = 500 units
Axles	500 − 250 = 250 units
Small Wheels	2(500) − 400 = 600 units

THE MRP PLANNING WORKSHEET

Although the net component requirements and order dates for the components have been calculated separately, the interaction between the two pieces of information over time is also important. One convenient way to record this information is on an MRP *planning worksheet* containing relevant information for each component over a given planning horizon. The MRP planning worksheet for a 10-period planning horizon is shown in Table CD6.6.

TABLE CD6.6 MRP Planning Worksheet

Item Name	Lead Time	Time Periods									
Inventory Rule	Safety Stock	1	2	3	4	5	6	7	8	9	10
Gross Requirements											
Scheduled Receipts											
Balance Available											
Net Requirements											
Planned Order Receipts											
Planned Order Releases											

The item name and known lead time for the item are entered in the upper left corner of the MRP Worksheet. The “inventory rule” entry provides information concerning requirements on the inventory order size. For some components, the firm may specify a *lot size* rule that designates a certain required order quantity. This quantity may be set to take advantage of quantity discounts or to simplify the production scheduling. For other components, the firm may use a *lot-for-lot* rule, by which the firm orders only the amount needed for a given period.

The "safety stock" entry designates the average inventory quantity the firm desires to keep in stock to cover losses due to shrinkage and to meet shortages that could result from unexpected delays in production or delivery of the component.

For each time period over the planning horizon,[1] the "gross requirements" for the component are determined from the net component requirements for the assemblies that use it. Because an MRP analysis is a dynamic process that uses a rolling time horizon, deliveries of the component may already be scheduled for some of the time periods in the planning horizon under analysis. These are accounted for in the "scheduled receipts" row of the worksheet.

The "balance available" row keeps a running total of the number of inventory units that will be available at the *end* of each time period. This number is equivalent to the balance available at the *beginning of the next period*. The entry to the left of the first period corresponds to the inventory balance at the *beginning* of the first period. The rest of the entries in this row are calculated after determining the net requirements row, the planned order receipt row, and the planned order release row.

"Net requirements" refer to the amount of the component required to satisfy the gross requirements and safety stock for the period after accounting for any scheduled receipts and are calculated by:

$$\begin{aligned}(\text{Net Requirements}) = {} & (\text{Gross Requirements}) + (\text{Safety Stock}) \\ & - (\text{Scheduled Receipts}) \\ & - (\text{Balance Available from Previous Time Period})\end{aligned}$$

If this calculation results in a negative difference (i.e., the scheduled receipts plus the balance available exceed the gross requirements plus the safety stock), then the net requirements are simply 0. If the net requirements for a given period are positive, "planned order receipts" of inventory are needed for that period. This amount should be at least equal to the net requirements. If a lot-for-lot rule is used for ordering the inventory, then the planned order receipts for the period are the net requirements for the period. If a lot size rule is used, the planned order receipts equal the smallest multiple of the lot size necessary to meet the net requirements.

The "planned order release" date takes into account the inventory lead time. For periods that carry planned order receipts, corresponding planned order releases must exist L periods earlier, (L being the inventory lead time). For example, if there is a planned order receipt of 800 units for a component in week 8 and the component has a lead time of three weeks, there must be a planned order release in week 5 (= 8 − 3).

The following formula can therefore be used to determine the entries in the balance available row of the MRP worksheet:

$$\begin{aligned}(\text{Balance Available at End of Period}) = {} & (\text{Balance Available at Beginning of Period}) \\ & - (\text{Gross Requirements}) + (\text{Scheduled and Planned Order Receipts})\end{aligned}$$

Given this background, the MRP analysis for the production of tricycles at Little Trykes can now be extended to a multiperiod planning horizon. Because the production process for making tricycles spans seven weeks, the planning begins seven weeks into the future, starting at week 8.

[1] In practice, the actual dates are used rather than the numerical time periods (i.e., week 1 might be June 11, week 2, June 18, etc.).

LITTLE TRYKES TOYS, INC. (CONTINUED)

Little Trykes Toys, Inc. has forecast the following demand for tricycles between weeks 8 and 16:

Week	8	9	10	11	12	13	14	15	16
Demand	90	75	85	100	80	60	70	80	95

Company policy is to produce tricycles in lot sizes of 250 and require 30 units of safety stock. The lead time for assembly of tricycles is two weeks.

Based on previous planning, LTT expects an available balance of 150 tricycles at the beginning of week 8, and the receipt of 250 tricycles is scheduled at the beginning of week 9. Management wishes to determine the MRP worksheet for tricycles in order to properly plan production during this time frame.

SOLUTION

Accounting for the demand data and other problem information, the initial MRP worksheet is shown in Table CD6.7.

TABLE CD6.7 Initial MRP Worksheet for Tricycles

Tricycles	Lead Time 2 Weeks		Time Periods								
Lot Size 250	Safety Stock 30		8	9	10	11	12	13	14	15	16
Gross Requirements			90	75	85	100	80	60	70	80	95
Scheduled Receipts				250							
Balance Available		150									
Net Requirements											
Planned Order Receipts											
Planned Order Releases											

The following analysis completes the tricycle MRP worksheet for balance available, net requirements, planned order receipts, and planned order releases on a week-by-week basis.

Week 8

Net Requirements: Although no receipts are scheduled for week 8, the balance available at the beginning of week 8 (150) exceeds the sum of the gross requirements for that week (90) plus the safety stock of 30. Hence, the net requirements for week 8 are 0, and no planned order receipts are required.

Balance Available at End of Week 8 (***Beginning of Week 9***)

$$\begin{pmatrix}\text{Balance Available}\\ \text{at End of Week 8}\end{pmatrix} = \begin{pmatrix}\text{Balance Available}\\ \text{at Beginning of Week 8}\end{pmatrix} - \begin{pmatrix}\text{Gross Requirements}\\ \text{in Week 8}\end{pmatrix} + \begin{pmatrix}\text{Scheduled and Planned}\\ \text{Order Receipts}\end{pmatrix}$$
$$= 150 - 90 + 0 = 60$$

This allows the MRP worksheet to be updated as in Table CD6.8.

TABLE CD6.8 MRP Worksheet for Tricycles—Week 8

Tricycles	Lead Time 2 Weeks		Time Periods								
Lot Size 250	Safety Stock 30		8	9	10	11	12	13	14	15	16
Gross Requirements			90	75	85	100	80	60	70	80	95
Scheduled Receipts				250							
Balance Available		150	60								
Net Requirements			0								
Planned Order Receipts											
Planned Order Releases											

Week 9

Net Requirements: The sum of the balance available at the beginning of the week (60) plus the scheduled receipts in week 9 (250) again exceeds the gross requirements (75) plus the safety stock (30). Hence, there are no net requirements.

Balance Available at End of Week 9 (***Beginning of Week 10***)

$$\begin{aligned}\begin{pmatrix}\text{Balance Available}\\ \text{at End of Week 9}\end{pmatrix} &= \begin{pmatrix}\text{Balance Available}\\ \text{at Beginning of Week 9}\end{pmatrix} - \begin{pmatrix}\text{Gross Requirements}\\ \text{in Week 9}\end{pmatrix}\\ &\quad + \begin{pmatrix}\text{Scheduled and Planned}\\ \text{Order Receipts}\end{pmatrix}\\ &= 60 - 75 + 250 = 235\end{aligned}$$

Weeks 10 and 11

Net Requirements: Net requirements for weeks 10 and 11 remain at 0.

Balance Available at End of Week: The balances available at the end of weeks 10 and 11 equal the balances available at the beginning of the respective weeks, reduced by the gross requirements of 85 and 100, respectively, giving rise to the worksheet in Table CD6.9.

TABLE CD6.9 MRP Worksheet for Tricycles—Through Week 11

Tricycles	Lead Time 2 Weeks		Time Periods								
Lot Size 250	Safety Stock 30		8	9	10	11	12	13	14	15	16
Gross Requirements			90	75	85	100	80	60	70	80	95
Scheduled Receipts				250							
Balance Available		150	60	235	150	50					
Net Requirements			0	0	0	0					
Planned Order Receipts											
Planned Order Releases											

Week 12

Net Requirements: In week 12, the gross requirements are 80, and the safety stock requirements are 30. Thus, there is a need for 110 units. Because the balance available from week 11 is only 50, however, the net requirements for week 12 are for 80 + 30 − 50 = 60 units.

Planned Order Receipts: Since LTT uses a production lot size of 250 units, there must be a planned order receipt of 250 for week 12.

Planned Order Releases: Given that the lead time for assembling the tricycles is two weeks, to meet a planned order receipt in week 12, there must be a planned order release of 250 in week 10 (12 − 2).

Balance Available at End of Week 12

$$\begin{pmatrix}\text{Balance Available}\\ \text{at End of Week 12}\end{pmatrix} = \begin{pmatrix}\text{Balance Available}\\ \text{at Beginning of Week 12}\end{pmatrix} - \begin{pmatrix}\text{Gross Requirements}\\ \text{for Week 12}\end{pmatrix} + \begin{pmatrix}\text{Planned Order Receipt}\\ \text{for Week 12}\end{pmatrix}$$
$$= 50 - 80 + 250 = 220 \text{ units}$$

The worksheet at this point is given in Table CD6.10.

TABLE CD6.10 MRP Worksheet for Tricycles—Through Week 12

Tricycles		Lead Time 2 Weeks	Time Periods								
Lot Size 250		Safety Stock 30	8	9	10	11	12	13	14	15	16
Gross Requirements			90	75	85	100	80	60	70	80	95
Scheduled Receipts				250							
Balance Available		150	60	235	150	50	220				
Net Requirements			0	0	0	0	60				
Planned Order Receipts							250				
Planned Order Releases					250						

Weeks 13 and 14

The gross requirements of 60 and 70 units for weeks 13 and 14, respectively, plus the 30 units of safety stock are covered by the available balances for these weeks. Thus, there are no net requirements during this period, as reflected in Table CD6.11.

Weeks 15 and 16

Net Requirements: In week 15, there are gross requirements for 80 units. Although the balance available at the beginning of week 15 is 90 units, this quantity is not sufficient to meet the requirements for week 15 owing to the desired safety stock of 30 units. Hence, the net requirements for week 15 are for 80 + 30 − 90 = 20 units.

TABLE CD6.11 MRP Worksheet for Tricycles—Through Week 14

Tricycles	Lead Time 2 Weeks	Time Periods								
Lot Size 250	Safety Stock 30	8	9	10	11	12	13	14	15	16
Gross Requirements		90	75	85	100	80	60	70	80	95
Scheduled Receipts			250							
Balance Available	150	60	235	150	50	220	160	90		
Net Requirements		0	0	0	0	60	0	0		
Planned Order Receipts						250				
Planned Order Releases				250						

Planned Order Receipts: Given that LTT uses a production lot size of 250 units, there is a planned order recipt of 250 for week 15.

Planned Order Releases: Since the lead time for assembling the tricycles is two weeks, for a planned order release to be available in week 15 there must be a planned order release of 250 in week 13 (15 − 2).

Balance Available at End of Week 15

$$\begin{pmatrix}\text{Balance Available}\\ \text{at End of Week 15}\end{pmatrix} = \begin{pmatrix}\text{Balance Available}\\ \text{at Beginning of Week 15}\end{pmatrix} - \begin{pmatrix}\text{Gross Requirements}\\ \text{for Week 15}\end{pmatrix} + \begin{pmatrix}\text{Planned Order Receipt}\\ \text{for Week 15}\end{pmatrix}$$
$$= 90 - 80 + 250 = 260 \text{ units}$$

Incorporating the gross requirements of 95 units for week 16 gives the completed MRP worksheet for week 8 through week 16 shown in Table CD6.12.

TABLE CD6.12 MRP Worksheet for Tricycles—Weeks 8–16

Tricycles	Lead Time 2 Weeks	Time Periods								
Lot Size 250	Safety Stock 30	8	9	10	11	12	13	14	15	16
Gross Requirements		90	75	85	100	80	60	70	80	95
Scheduled Receipts			250							
Balance Available	150	60	235	150	50	220	160	90	260	165
Net Requirements		0	0	0	0	60	0	0	20	0
Planned Order Receipts						250			250	
Planned Order Releases				250			250			

MRP PLANNING WORKSHEET FOR SUBCOMPONENTS

Once the planned order releases have been determined for one hierarchical level, the MRP planning worksheets for the next lower hierarchical level can be generated. For example, in the Little Trykes problem, once the planned order releases for tricycles (hierarchical level 0) have been determined, the MRP worksheets for all hierarchical level 1 components (frames, seats, front wheel assemblies, and small wheel sets) can be generated. Because of space limitations, this process is illustrated only for the small wheel sets. Note that, since the lead time for small wheel sets is two weeks, the MRP planning worksheet for small wheel sets covers a period two weeks earlier than that considered for the tricycles, namely, week 6 through week 14.

LITTLE TRYKES TOYS, INC. (CONTINUED) SMALL WHEEL SETS

Little Trykes Toys, Inc. uses a lot-for-lot inventory rule for producing the small wheel sets. It requires a safety stock of 50 sets and projects that a balance of 80 small wheel sets will be available at the beginning of week 6. Management would like to develop an MRP worksheet for the small wheel sets used in the production of the tricycles over the nine-week planning period.

SOLUTION

Based on the MRP planning worksheet for the tricycle production, 250 small wheel sets must be available by weeks 10 and 13 in order for Little Trykes to support the planned order releases for the tricycles. This quantity becomes the gross requirements for the small wheel sets. (Note that the 250 small wheel sets required in week 7 to support the scheduled receipt of 250 tricycles in week 9 are excluded from the MRP planning worksheet because this amount would have been previously accounted for in an earlier planning horizon.) The initial worksheet is given in Table CD6.13.

TABLE CD6.13 Initial MRP Worksheet for Small Wheel Sets

Small Wheel Sets	Lead Time 2 Weeks		Time Periods								
Lot for Lot	Safety Stock 50		6	7	8	9	10	11	12	13	14
Gross Requirements			0	0	0	0	250	0	0	250	0
Scheduled Receipts											
Balance Available		80									
Net Requirements											
Planned Order Receipts											
Planned Order Releases											

Week 6 Through Week 9

Because the gross requirements for week 6 through 9 are 0, net requirements and planned order receipts are 0 for this time period, and the balance available remains at 80.

Week 10

Net Requirements: The gross requirements of 250 units imply net requirements of $250 + 50 - 80 = 220$.

Planned Order Receipts: Because a lot-for-lot rule is used for the small wheel sets, a planned order receipt equal to the net requirements of 220 for week 10 is required.

Planned Order Releases: Because there is a two-week lead time for small wheel sets, a planned order release of 220 is scheduled for week 8 (=10 − 2).

Balance Available

$$\begin{pmatrix}\text{Balance Available}\\ \text{at End of Week 10}\end{pmatrix} = \begin{pmatrix}\text{Balance Available}\\ \text{at Beginning of Week 10}\end{pmatrix} - \begin{pmatrix}\text{Gross Requirements}\\ \text{for Week 10}\end{pmatrix} + \begin{pmatrix}\text{Planned Order Receipt}\\ \text{for Week 10}\end{pmatrix}$$
$$= 80 - 250 + 220 = 50 \text{ units}$$

This is the required safety stock. At this point, the MRP planning worksheet for small wheel sets is as shown in Table CD6.14.

Table CD6.14 MRP Worksheet for Small Wheel Sets—Through Week 10

Small Wheel Sets	Lead Time 2 Weeks		Time Periods								
Lot for Lot	Safety Stock 50		6	7	8	9	10	11	12	13	14
Gross Requirements			0	0	0	0	250	0	0	250	0
Scheduled Receipts											
Balance Available		80	80	80	80	80	50				
Net Requirements							220				
Planned Order Receipts							220				
Planned Order Releases					220						

Week 11 Through Week 14

The balance available stays at 50 through the remaining time periods. In week 13, the gross demand of 250 units implies net requirements of 250 units. Hence, there must be a planned order receipt of 250 in week 13 and a planned order release of 250 in week 11 (=13 − 2). The completed worksheet for small wheel sets is given in Table CD6.15.

Table CD6.15 Completed Worksheet for Small Wheel Sets—Weeks 6–14

Small Wheel Sets	Lead Time 2 Weeks		Time Periods								
Lot for Lot	Safety Stock 50		6	7	8	9	10	11	12	13	14
Gross Requirements			0	0	0	0	250	0	0	250	0
Scheduled Receipts											
Balance Available		80	80	80	80	80	50	50	50	50	50
Net Requirements							220			250	
Planned Order Receipts							220			250	
Planned Order Releases					220			250			

A similar process is used for hierarchical level 2 components. To illustrate, consider the small wheels that go into the small wheel sets.

LITTLE TRYKES TOYS, INC. (CONTINUED) SMALL WHEELS

Management wishes to determine the MRP planning worksheet for small wheels over the nine-week planning period between week 3 and week 11. The company has determined that it needs a safety stock of 200 small wheels; in order to obtain quantity discounts from its supplier, it orders small wheels in lots of 800. A balance of 225 wheels is expected to be available at the beginning of week 3.

SOLUTION

Recall that each small wheel set requires two small wheels and that the lead time for delivery of the small wheels is three weeks. Hence, the appropriate time frame for the MRP planning worksheet is from week 3 through week 11. Table CD6.16 shows the completed MRP worksheet for this component.

TABLE CD6.16 Completed MRP Worksheet for Small Wheels

Small Wheels	Lead Time 3 Weeks		Time Periods								
Lot Size 800	Safety Stock 200		3	4	5	6	7	8	9	10	11
Gross Requirements			0	0	0	0	0	440	0	0	500
Scheduled Receipts											
Balance Available		225	225	225	225	225	225	585	585	585	885
Net Requirements								415			115
Planned Order Receipts								800			800
Planned Order Releases					800			800			

The inventory planning worksheets for each of the other components making up the tricycle can be determined in a similar manner.

GOODS SHARING THE SAME COMPONENTS

One important aspect of the MRP process that has been ignored thus far is the impact on inventory of several finished goods sharing the same components. For LTT, this occurs with the small wheel sets, used both in tricycles and wagons. To illustrate, consider the effect of wagon demand on the MRP worksheet for the small wheel sets.

LITTLE TRYKES TOYS, INC. (CONTINUED) WAGONS AND SMALL WHEEL SETS

LTT has forecasted the following demand for wagons during the period from week 7 through 15:[2]

Week	7	8	9	10	11	12	13	14	15
Demand	80	55	75	110	90	70	100	90	95

[2] A one-week period earlier than that used for the tricycles is considered because the lead time for wagon assembly is one week less than that for tricycles.

Wagons are produced in lot sizes of 300 and require no safety stock. The company would like to determine the MRP worksheet for the wagons during this period, given a projected balance of 140 units at the beginning of week 7. It would also like to investigate the effect this demand will have on the MRP worksheet for the small wheel sets.

SOLUTION

Applying the same analysis as that performed for tricycles results in the completed MRP planning worksheet for wagons given in Table CD6.17.

TABLE CD6.17 Completed MRP Worksheet for Wagons

Wagons	Lead Time 1 Week	Time Periods								
Lot Size 300	Safety Stock 0	7	8	9	10	11	12	13	14	15
Gross Requirements		80	55	75	110	90	70	100	90	95
Scheduled Receipts										
Balance Available	140	60	5	230	120	30	260	160	70	275
Net Requirements		0		70	0	0	40	0	0	25
Planned Order Receipts				300			300			300
Planned Order Releases			300			300			300	

As can be seen from the planned order releases, 300 small wheel sets are required for wagon production in weeks 8, 11, and 14. Recall that it was previously determined that 250 small wheel sets are required in weeks 10 and 13 to support tricycle production over its planning period. Incorporating this information into the planning worksheet gives the revised MRP planning worksheet for the small wheel sets shown in Table CD6.18. This new worksheet differs from the MRP planning worksheet for small wheel sets in Table CD6.15 due to the additional gross requirements of 300 units in weeks 8, 11, and 14.

TABLE CD6.18 Revised MRP Worksheet for Small Wheel Sets

Small Wheel Sets	Lead Time 2 Weeks	Time Periods								
Lot For Lot	Safety Stock 50	6	7	8	9	10	11	12	13	14
Gross Requirements		0	0	300	0	250	300	0	250	300
Scheduled Receipts										
Balance Available	80	80	80	50	50	50	50	50	50	50
Net Requirements			0	270		250	300		250	300
Planned Order Receipts				270		250	300		250	300
Planned Order Releases		270		250	300		250	300		

Analysis of Orders

Over the nine-week period, five orders will be placed for small wheel sets if Little Trykes uses a lot-for-lot rule in producing these units. In particular, orders for 300 wheel sets immediately follow the orders for 250 wheel sets. On a practical basis, this does not seem particularly efficient.

One way of avoiding these extra production setups is to specify a given production lot size. Because demand for wheel sets is not constant over time, however, such a strategy will not necessarily yield an optimal policy. The next section, addresses the problem of determining the inventory order policy in situations in which future demand forecasts vary over time.

II Inventory Models with Time Varying Demand

In Chapter 8 it was shown that, when demand is reasonably constant over time, the EOQ model may be used to determine an optimal inventory policy. If demand varies greatly from period to period, however, an EOQ solution may be far from optimal.

Demands can vary significantly over time because of fluctuations in consumer preferences. Even when demand for a finished good does not vary significantly, as shown earlier, component demand need not be stationary. The **Wagner-Whitin algorithm**[3] can be used to determine the optimal inventory policy if the following assumptions hold:

1. Demand is known for each period over some time horizon.
2. While demand may vary from period to period, within each period, demand occurs at a relatively constant rate.
3. If an order is placed for arrival during a given period, it arrives at the beginning of the delivery period.
4. Holding costs are incurred at a known fixed rate. The holding cost for a period is a function of the average inventory level during that period.

Although assumptions 2 and 4 are not critical to developing the inventory model, they are included in the set of assumptions to clarify how the inventory holding costs are being incurred.

THE WAGNER-WHITIN ALGORITHM

The Wagner-Whitin algorithm uses a dynamic programming technique to determine an optimal inventory policy. As discussed in Chapter 13 on this CD-ROM, dynamic programming is a family of management science techniques that takes advantage of the fact that certain problems can be solved by breaking them into smaller, more manageable subproblems.

The Wagner-Whitin algorithm makes use of two important concepts:

1. If there is an order in some period, an optimal inventory policy orders an amount exactly equal to the sum of the next k period's future demands (where k is an integer determined by the algorithm).
2. There is an upper limit to the value of k.

To illustrate these two concepts, consider the situation faced by Golden West Homes, a manufacturer of modular housing.

[3] Named for Harvey Wagner and Thomas Whitin, who developed the algorithm in 1958.

GOLDEN WEST HOMES

Golden West Homes purchases kitchen ranges for its homes from Admiral Appliance Company at a cost of $325 each. Golden West uses a 16% annual holding cost rate, and the cost to place an order with Admiral is $100. Golden West forecasts the following requirements for Admiral kitchen ranges over the next 10 weeks:

Week	1	2	3	4	5	6	7	8	9	10
Demand	120	40	80	60	90	30	20	90	60	130

Management would like to determine the optimal ordering policy for the ranges over this planning horizon.

SOLUTION

According to concept 1, if an order is placed to arrive at the beginning of week 1, it is for either 120 units (to satisfy the demand for week 1), 160 units (to satisfy the demand for weeks 1 and 2), 240 units (to satisfy the demand for week 1 through week 3), and so on. An order of say, 200, would *not* be placed since it would cover demand for two *and a half* weeks.

Because Golden West uses a 16% annual holding cost rate and the ranges cost the company $325 each, the annual holding cost for the SKU is C_h = .16($325) = $52. Thus, the weekly holding cost per unit is $1. In period 1, if 120 units are ordered (to satisfy the demand for that period only), the average inventory is 120/2 = 60. The total variable inventory cost (the sum of the order cost and the holding cost) is therefore $100 + 60($1) = $160.

If Golden West orders enough ranges to satisfy the demand for weeks 1 and 2 (160), then the inventory level at the beginning of week 1 would be 160, decrease to 40 (=160 − 120) by the end of that week, and further decrease from 40 to 0 during week 2. Hence, the average inventory level during week 1 is 100 units [= (160 + 40)/2], and, during week 2, it is 20 [= (40 + 0)/2] units. Figure CD6.6 shows the inventory profile of ordering 160 units in week 1. The cost of supplying inventory to cover the demand for weeks 1 and 2 is the sum of the $100 order cost, the holding cost for week 1 (100 ∗ $1), and the holding cost for week 2 (20 ∗ $1), a total of $220.

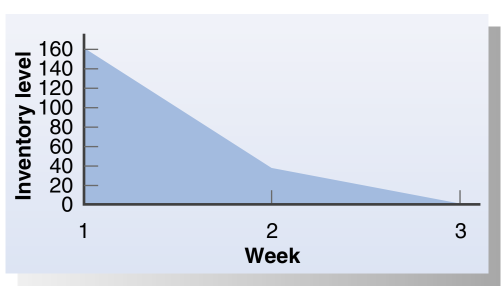


FIGURE CD6.6
Inventory Profile—over Two Weeks

Now consider ordering enough inventory in week 1 to satisfy the first three weeks of demand, 240 ranges. If an extra 80 ranges are ordered to satisfy the demand for week 3, however, these 80 units have to be kept in inventory for a full two weeks, costing an extra $160 (= $2 ∗ 80). Since it costs only $100 to place an order, it is clearly cheaper to delay ordering these 80 units until week 3. Hence, in week 1, it does not pay to order inventory to satisfy the demand beyond week 2. This is an illustration of concept 2 of the algorithm.

Summarizing, the following rule applies:

> If the marginal cost of ordering inventory for an extra week exceeds the cost of placing an order, it is not economical to place an order for that week or beyond.

Repeating this analysis for week 2 through week 10 gives the values found in Table CD6.19. The formulas for obtaining these values are given in Appendix CD6.1. The values, designated T(j, j + k), represent the cost of ordering an amount for delivery in week j (the row value) sufficient to satisfy the demand up to the beginning of week j + k (the column value). For example, the cost of ordering inventory for delivery in week 5, which is sufficient to last up to the beginning of week 7, is $190.

TABLE CD6.19 Inventory Analysis—Golden West Homes

Week	Order up to the Beginning of Week (j + k):									
(j)	2	3	4	5	6	7	8	9	10	11
1	160	220								
2		120	240							
3			140	230						
4				130	265	340	410			
5					145	190	240			
6						115	145			
7							110	245		
8								140	230	
9									130	
10										165

Once the "order up to" inventory cost values, T(j, j + k) have been calculated, dynamic programming can be used to determine the optimal inventory ordering policy over the time frame in question. One way of doing this is by constructing a network, with nodes (1, 2, 3, . . . , n) corresponding to the n time periods and the arcs between nodes j and j + k equal in value to T(j, j + k). The optimal inventory policy can then be found by identifying the shortest path through this network. In particular, if the shortest path contains an arc from period j to period j + k, an order will be placed for delivery at period j that is of sufficient size to handle the demand *up to, but not including, period j + k*.

Figure CD6.7 shows the network corresponding to this problem. Using the shortest path approach developed in Chapter 4, the shortest path through this network begins at node 1 and proceeds through nodes 3, 5, 8, and 10. Thus, the optimal inventory policy for Golden West Homes is as follows:

Week	*Order Amount*	*Order for Weeks*
1	120 + 40 = 160	1, 2
3	80 + 60 = 140	3, 4
5	90 + 30 + 20 = 140	5, 6, 7
8	90 + 60 = 150	8, 9
10	130	10

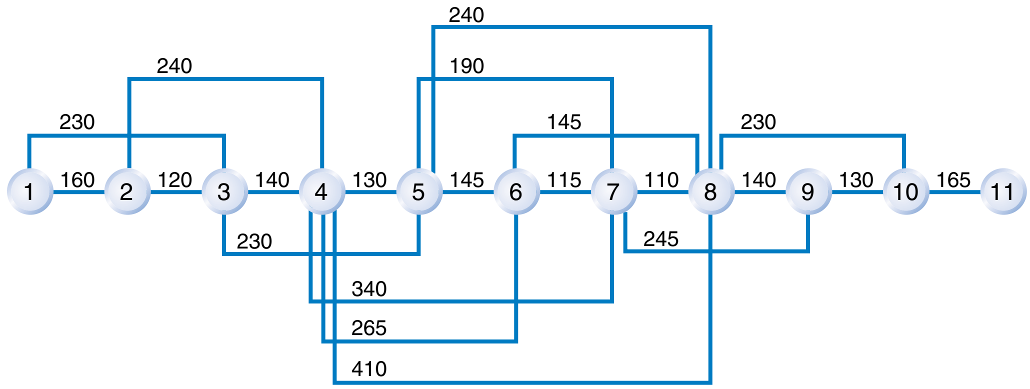


FIGURE CD6.7 Inventory Cost Network for Golden West Homes

The total cost of this policy is $1085. Figure CD6.8 summarizes the inventory profile corresponding to this solution over the 10-week (70-day) period.

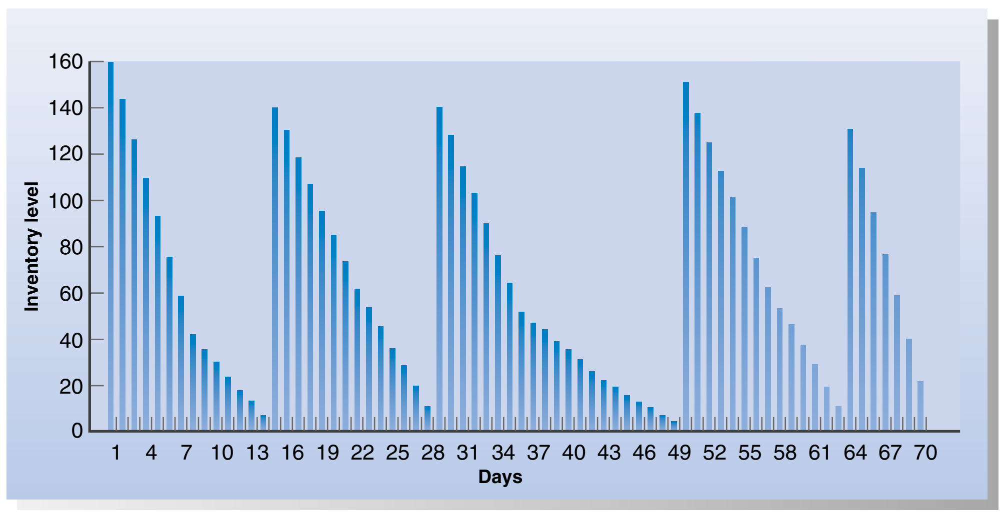


FIGURE CD6.8 Inventory Profile for Golden West Homes Over 10-Week Period

It is interesting to contrast the recommended solution to the EOQ solution. The average weekly demand over the 10-week period is 72 units. If this value is used for the weekly demand, D, then, since C_h is $1/week and C_o is $100, the EOQ order quantity is:

$$Q^* = \sqrt{\frac{2(72)\,(100)}{1}} = 120$$

Using an order quantity of 120 units, the firm would have to place six orders (in weeks 1, 2, 4, 5, 7, and 9), resulting in the inventory profile given in Figure CD6.9. The average weekly inventory level under this policy is given in Table CD6.20.

Since C_h is $1/per item per week, the sum of the holding costs over the 10-week period under the EOQ policy is $60 + $100 + $40 + . . . + $65 = $770. Adding the cost of placing six orders ($600) to the inventory holding cost ($770) gives a total inventory cost of $1370 over the 10-week period; this is $285 (or 26%) greater than the solution found using the Wagner-Whitin algorithm.

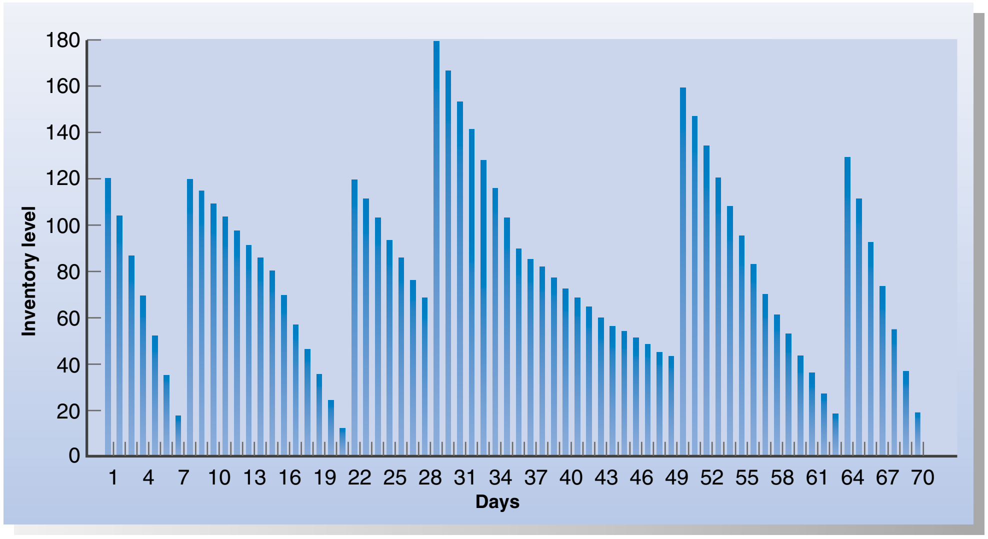


FIGURE CD6.9 Inventory Profile Based on 120-Unit Order Quantity (Six Orders)

TABLE CD6.20 Average Weekly Inventory Level—Golden West Homes

Week	1	2	3	4	5	6	7	8	9	10
Average Inventory Level	60	100	40	90	135	75	50	115	40	65

THE SILVER-MEAL HEURISTIC

The Wagner-Whitin algorithm gives the lowest-cost solution for an inventory problem when the forecasted demand is known but varies over time. Yet the solution is far more difficult to obtain than simply using the EOQ formula. To reduce the computational complexities of the Wagner-Whitin algorithm, management scientists began looking for a heuristic solution technique that was easier to implement and would give an answer close to optimal. One such technique is the **Silver-Meal heuristic**,[4] which utilizes the same cost data, T(j, j + k), as that used in the Wagner-Whitin algorithm but avoids the need to solve a shortest path network model. Instead, the procedure uses the following steps.

Silver-Meal Heuristic

Start at Period 1.

1. Calculate the per period average inventory holding and ordering cost of satisfying demand for the next k periods.
2. Order enough inventory to satisfy demand out to the period that has the lowest average cost.
3. If demand for the final period has not been satisfied, return to step (1) using the period found in step (2).

The Silver-Meal heuristic is illustrated using the cost data from the Golden West Homes example. In week 1, satisfying demand for one week costs $160, while satisfying demand for two weeks costs $220 (see Table CD6.19). Hence, the

[4] Named for Edward Silver and Harlan Meal, who proposed this technique in 1973.

per period average inventory cost of satisfying demand for one period is \$160 and for two periods \$110 (=\$220/2). Since \$110 is less than \$160, demand is satisfied for two weeks. The heuristic then returns to step (1) beginning at week 3. (As discussed earlier, one does not have to look beyond two weeks because the marginal cost of carrying inventory for additional weeks exceeds the cost of placing an order.)

From Table CD6.19 it is seen that satisfying demand for week 3 only gives a per period average inventory cost of \$140, while satisfying demand for weeks 3 and 4 gives a per period average inventory cost of \$115 (=\$230/2). Hence, Golden West Homes should place an order in week 3 which satisfies demand for two more weeks. The heuristic then returns to step (1) beginning at week 5.

Continuing in this fashion using the data from Table CD6.19, gives:

	Number of Periods	*Average per Period Cost*
	1	\$145/1 = \$145
Week 5	2	\$190/2 = \$95
	3	\$240/3 = \$80

Satisfy demand for the next three periods

	Number of Periods	*Average per Period Cost*
	1	\$140/1 = \$140
Week 8	2	\$230/2 = \$115

Satisfy demand for the next two periods

	Number of Periods	*Average per Period Cost*
Week 10	1	\$165/1 = \$165

Satisfy demand for this period only.

In this case, the Silver-Meal heuristic indicates that Golden West Homes should place orders in periods 1, 3, 5, 8, and 10. This is exactly the same solution achieved with the Wagner-Whitin algorithm.

Although the Silver-Meal heuristic may not always yield the same solution as the Wagner-Whitin algorithm, in general, it gives a solution that either is optimal or differs in cost from the optimal solution by only a few percent.

III Current Trends in Inventory Control

JUST-IN-TIME SYSTEMS

In recent years, management scientists have paid a great deal of attention to Japanese management techniques. An inventory/production control technique pioneered in Japan, known as **just-in-time**, or **JIT**, can reduce work-in-process inventories to the lowest possible level by keeping production lot sizes small. Although an MRP or EOQ inventory control system can also utilize small production lot sizes, JIT differentiates the way in which production decisions are made and in the movement of inventory from one hierarchical level to another.

Traditional inventory systems, such as MRP, are referred to as *push systems*. Decisions are made regarding the production schedule for the subcomponents based on the forecast demand for the finished good. Once these subcomponents have been produced, they are "pushed up" to the next level of assembly. By contrast, a just-in-time system is characterized as a *pull system*. A subcomponent is produced only on request from the work center that utilizes the subcomponent in assembly. Hence, the inventory is "pulled" through the system.

The size of the lot that is pulled from one hierarchical level to the next is intentionally kept small, with a minimum buildup of safety or buffer stock. Consequently, if there is a failure in the production process at some point in the system, the entire production system may have to shut down. This is because once production of a component ceases two things happen. First, the assemblies that utilize the component quickly run out of inventory for the component and have to stop production. Second, the assemblies that go into making the component quickly find themselves with a sufficient inventory buildup and, therefore, also have to stop production.

Although it may appear quite inefficient that an entire system shuts down whenever there is a problem with the production of a single component, this policy immediately focuses attention on the problem area. As a result, great efforts are made to correct problems quickly. Workers typically learn to handle more than one task so that labor necessary to correct production deficiencies is readily available and shut-downs usually do not last long. The necessity of avoiding problems and quickly solving whatever difficulties arise results in a general improvement in total product quality.

For a just-in-time system to operate properly, the factory and its suppliers must be completely coordinated. Because work stoppages can result in idling the entire factory, the quality of the components supplied by vendors is of crucial importance. In addition, the small lot sizes require frequent deliveries on a reliable schedule. Accordingly, a factory normally selects only one supplier for a particular component. This *single sourcing* allows the supplier and the factory to develop a full partnership in the production process. On the downside, the factory is left vulnerable to any production problems that might occur with the supplier. From the supplier's standpoint, it can be dangerous to rely too greatly on a single customer because downturns in customer demand can have a devastating effect on the supplier's profitability. Perhaps for these reasons, JIT has not yet won the same degree of acceptance in the United States as it has in Japan.

KANBAN SYSTEMS

The ordering of inventory from one production level to the next in JIT systems is frequently done using a **kanban** system. This is a ticket-based system (*kanban* is the Japanese word for these tickets) that keeps track of the flow of components through the factory as described below.

Kanban System

1. A component is produced in a small batch, each of which is attached to a *production ordering kanban*. The batches are then shipped to an intermediary storage location known as the *store*, where they are held until needed for production in some assembly.
2. The assembly that utilizes this component collects the component from the store and brings it to a *holding area*. When the component is picked up from the store, a worker removes the production ordering kanbans from the component and returns them to the component production area. At the same time, the worker attaches a *withdrawal kanban* to the component.
3. When the component is removed from the holding area for use in production, the withdrawal kanban is collected and placed on a *kanban post*. When the number of withdrawal kanbans on the kanban post reaches a sufficient level, a worker returns to the store with the withdrawal kanbans to initiate another delivery to the holding area.
4. The production ordering kanbans removed from the component at the store and returned to the component production area are also placed on a kanban post. When the number of production ordering kanbans on this post reaches a predetermined amount, production of the component is resumed.

Management science techniques can be used to determine optimal batch sizes and the optimal number of kanbans required to trigger a resumption of production or a visit to the store to obtain additional inventory.

Although kanbans are typically used in JIT applications, they are not essential. As the cost of computing power decreases, one should expect to see the functions performed by kanbans increasingly taken over by computers. An integrated computer control system is able to identify problem areas more quickly and allow dynamic determination of optimal production lot sizes and safety stocks.

FLEXIBLE MANUFACTURING SYSTEMS

Another production/inventory approach that is gaining popularity is the notion of a **flexible manufacturing system**, or **FMS**, in which computer-controlled machines are capable of performing several different operations used in the production process. The goods to be manufactured are placed on pallets, and their movements in the factory are controlled by a computer. Because a single machine may have many different functions, an item may visit a particular machine on more than one occasion during the manufacturing process.

Flexible manufacturing systems are most appropriate in factories that produce a moderate variety of goods, none of which has the sales volume to warrant mass production. By necessity, retooling and setup times for the machines have to be short enough that machine efficiency is not sacrificed. Management science techniques such as queuing theory (see Chapter 9) have been used in flexible manufacturing systems to determine the optimal movement of a good among the different machines making up the manufacturing process.

A flexible manufacturing system offers several advantages over a conventional assembly-line system in which each machine has a dedicated function. These include: (1) reduced work-in-progress inventory, (2) reduced manufacturing lead times, (3) reduced labor costs, (4) reduced plant space requirements, (5) increased machine utilization, and (6) increased number of different products the factory can produce.

Although these advantages are substantial, flexible manufacturing systems are far more costly than traditional assembly-line systems because of the higher cost of the sophisticated machinery required. Hence, even though FMS has found acceptance in some industries (such as metal-working), the cost barrier has limited the degree to which this technology has been embraced. As costs decrease, however, one should see an increase in the use of these systems.

APPENDIX CD6.1

Calculating Costs of Satisfying k Periods Worth or Demand for Inventory Models with Time Varying Demand

Define:

$T(j, j + k)$ = the inventory holding and ordering cost of placing an order for delivery in period j that will satisfy the demand for the SKU up to, but not including, period j + k

Y = number of periods per year

d_i = demand in period i

C_h = annual holding cost per unit

C_o = cost of placing an order

The formula for T(j, j + k) is:

$$T(j, j+k) = \left(\frac{C_h}{Y}\right)\sum_{i=1}^{k}\frac{(2i-1)d_{j+i-1}}{2} + C_o$$

For each time period j, one can stop calculating T(j, j + k) for values of k in which $T(j, j + k + 1) > T(j, j + k) + C_o$. This is equivalent to stopping the calculation of the T(j, j + k) values at the lowest value of k for which the following relationship is satisfied:

$$\left(\frac{C_h}{Y}\right)(2k-1)d_{j+k-1} > 2C_o$$

To illustrate the use of the above formulas consider period 2 in the Golden West Homes example in Section II of Supplement CD6. Recall that Golden West wished to determine the optimal policy for procurement of Admiral kitchen ranges over a 10-week period. Then,

$$T(2, 3) = \left(\frac{C_h}{Y}\right)\sum_{i=1}^{1}\frac{(2i-1)d_{1+i}}{2} + C_o = \left(\frac{52}{52}\right)\frac{d_2}{2} + C_o = \frac{40}{2} + 100 = 120$$

and

$$T(2, 4) = \left(\frac{C_h}{Y}\right)\sum_{i=1}^{2}\frac{(2i-1)d_{1+i}}{2} + C_o = \left(\frac{52}{52}\right)\left(\frac{d_2}{2} + \frac{3d_3}{2}\right) + C_o = 240$$

Additional values for T(2, 2 + k) do not have to be calculated since when k = 3, $5d_4 = 300 > 200$.

Problems

1. Consider Little Trykes Toys' production of wagons during week 16 through week 23, discussed in Section II. Suppose Little Trykes management forecasts the following demand for the wagons:

Week	16	17	18	19	20	21	22	23
Demand	65	120	95	80	140	170	125	80

Recall that wagons have a one-week lead time. Little Trykes uses a production lot size of 300, and no safety stock is desired. If Little Trykes has a balance available of 200 wagons at the beginning of week 16, determine the MRP planning worksheet for the period from week 16 through week 23.

2. Consider the following MRP planning worksheet for Little Trykes tricycles for week 8 through week 16:

Tricycles	Lead Time 2 Weeks		Time Periods								
Lot Size 250	Safety Stock 30		8	9	10	11	12	13	14	15	16
Gross Requirements			90	75	85	100	80	60	70	80	95
Scheduled Receipts				250							
Balance Available		150	60	235	150	50	220	160	90	260	165
Net Requirements			0	0	0	0	60	0	0	20	0
Planned Order Receipts							250			250	
Planned Order Releases					250			250			

Little Trykes produces front wheel assemblies in lot sizes of 200 units and desires a safety stock of 60 units. There is a scheduled receipt of 200 front wheel sets in week 10, and a balance of 90 front wheel assemblies is available at the beginning of week 7. Recall that the lead time for assembling front wheel assemblies is one week. Using this information, determine the MRP planning worksheet for front wheel assemblies for week 7 through week 15.

3. Golden West Homes manufactures mobile homes. Requirements for ovens over a 10-week planning period are as follows:

Week	1	2	3	4	5	6	7	8	9	10
Demand	130	160	60	50	40	80	120	140	70	150

The company purchases the ovens from the Swiss Maid Appliance Company. The estimated cost to place an order with Swiss Maid is $800, and Golden West estimates the weekly holding cost at $2 per oven. The company had been using the EOQ model to determine its ordering policy but is considering using the Wagner-Whitin algorithm instead. Golden West management has calculated the following values for $T(j, j + k)$, the inventory cost of ordering in week j to supply demand up to the beginning of week $j + k$:

Week	Order up to the Beginning of Week j + k									
j	2	3	4	5	6	7	8	9	10	11
1	930	1410	1710	2060	2420					
2		960	1140	1390	1670	2390				
3			860	1010	1210	1770				
4				850	970	1370				
5					840	1080	1680			
6						880	1240	1940	2430	
7							920	1340	1690	
8								940	1150	1900
9									870	1320
10										950

a. If Golden West forecasts its annual demand based on a simple 10-week moving average, determine the EOQ value for ovens.
b. If Golden West uses the Wagner-Whitin algorithm, what is the optimal inventory policy for ovens over the 10-week period?
c. If Golden West uses the Silver-Meal heuristic, what is the inventory policy for ovens over the 10-week period?

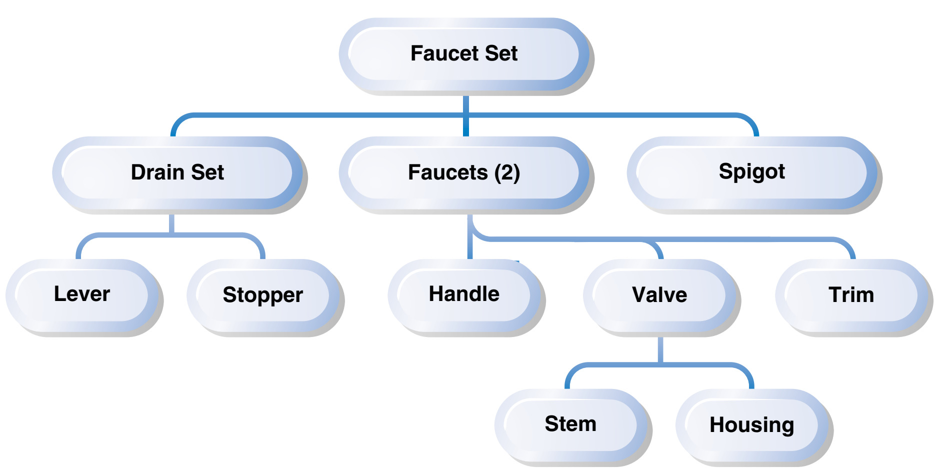
4. Consider the data given in problem 3. Determine the total inventory cost over the 10-week period for
a. the EOQ policy
b. the Wagner-Whitin policy
c. the Silver-Meal policy

5. Little Trykes is considering importing a children's pedal car to sell under the Little Trykes brand name. The company estimates the demand for the cars over the next eight-week planning period to be as follows:

Week	1	2	3	4	5	6	7	8
Demand	210	160	190	80	300	130	110	260

The cost to order the cars is an estimated $160, and the annual holding cost per car is approximately $13 (weekly cost of $0.25).
a. Determine the inventory policy for the cars over this eight-week period using the Silver-Meal heuristic.
b. How much cheaper is the Silver-Meal inventory policy than the EOQ policy over the eight-week period? (Assume that an eight-week simple moving average is used to forecast annual demand.)

6. Flavan Faucet manufactures a designer line of bathroom faucets. The following product tree represents the bill of materials for the Athena 242 faucet set.



The lead time and current inventory for each component are as follows.

Item	*Lead Time (in weeks)*	*Current Inventory*
Faucet Set	1	200
Drain Set	3	300
Lever	4	500
Stopper	3	400
Faucets	2	500
Handle	6	100
Valve	3	500
Stem	5	800
Housing	2	300
Trim	7	1200
Spigot	7	600

Suppose the company wishes to schedule production for 1000 units of the Athena 242 faucet set and wants the production completed in 10 weeks. Determine the order quantity and the latest date the order can be initiated for each component of the faucets.

7. Consider Flavan Faucet's production of Athena 242 faucet sets in problem 6. Suppose the company forecast the following demand between week 12 and week 18:

Week	12	13	14	15	16	17	18
Demand	800	600	400	200	600	400	500

The company estimates that the production setup cost to produce the Athena 242 faucet is $1400 and the annual holding cost per unit is $12. Flavan Faucet has a safety stock requirement of 200 faucets and a balance of 200 faucets available at the beginning of week 12. If Flavan Faucet uses the EOQ formula to determine the production lot size, determine the MRP planning worksheet over the period from week 11 through week 18. (Assume that annual demand is forecast based on a seven-week simple moving average and that demand in week 11 has already been addressed through previous analysis and is 0.)

8. Consider the data of problem 7. Suppose Flavan Faucet uses the Silver-Meal heuristic to schedule production of the faucet sets. Determine the MRP planning worksheet over the period from week 11 through week 18 in this case. (Assume that demand in week 11 has already been addressed through previous analysis and is 0.)

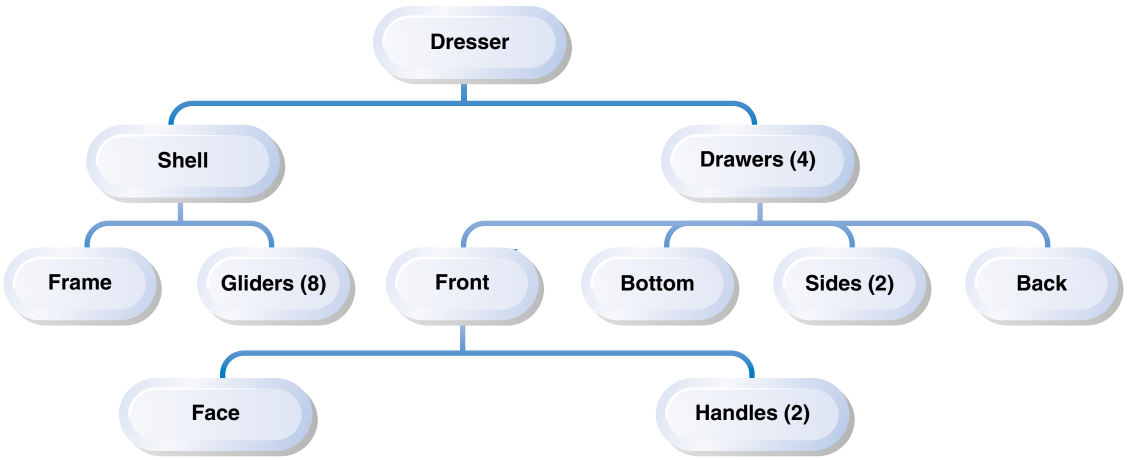
9. Consider the data of problem 7. If Flavan Faucet uses a lot-for-lot rule to schedule faucet set production, the MRP planning worksheet for week 11 through week 18 is as follows.

Faucet Sets	Lead Time 1 Week		Time Periods							
Lot for Lot	Safety Stock 200		11	12	13	14	15	16	17	18
Gross Requirements			0	800	600	400	200	600	400	500
Scheduled Receipts										
Balance Available		200	200	200	200	200	200	200	200	200
Net Requirements				800	600	400	200	600	400	500
Planned Order Receipts			800	600	400	200	600	400	500	
Planned Order Releases		800	600	400	200	600	400	500		

Recall that the lead time for the drain sets is three weeks. Flavan Faucet uses a production lot size of 1200 for the drain sets and desires a safety stock of 400 units. If there is a balance available of 500 drain sets at the end of week 10 and a scheduled receipt of 200 drain sets in week 11, determine the MRP planning worksheet for drain sets during the period from week 8 through week 17.

10. Consider the data from problem 9. Recall that the lead time for faucets is two weeks. Flavan Faucet is considering using a production lot size of 3000 for the faucets with no safety stock. If no balance is available at the end of week 10, but there is a scheduled receipt of 3000 faucets in week 11, determine the MRP planning worksheet for faucets during the period from week 9 through week 17.

11. Barnett Furniture Company (BFC) manufactures bedroom furniture for sale through mass merchandise retailers. One of its more popular products is a white laminate four-drawer dresser. The bill of materials for this dresser is represented by the following product tree.



The company has just received a rush order for 2500 of these dressers. Lead times and the current inventory position of each component are as follows.

Item	*Lead Time (in weeks)*	*Current Inventory*
Dresser	5	300
Shell	5	50
Frame	10	100
Gliders	15	1600
Drawers	10	0
Front	5	200
Face	5	600
Handles	10	24000
Bottom	10	800
Sides	10	4000
Backs	5	3000

a. If Barnett wants to complete the order in 40 days, determine how many of each component should be ordered and the latest possible day to order each component.
b. What is the shortest amount of time required for Barnett to complete this order?

12. Barnett Furniture Company (see Problem 11) is open five days a week. The lead time to complete the final assembly of white laminate four-drawer dressers is one week. BFC desires a safety stock of 200 dressers and produces in lot sizes of 2400 units. The company has forecast the demand over the planning period from week 7 through week 13 as follows.

Week	7	8	9	10	11	12	13
Demand	500	1000	1500	3000	1800	2200	1800

If BFC has a balance available of 100 dressers at the end of week 6 and a scheduled receipt of 2400 dressers in week 7, determine the MRP planning worksheet for dressers during the period from week 7 through week 13.

13. Consider the situation at Barnett Furniture Company given in problems 11 and 12. Based on an MRP analysis, BFC estimates the following gross requirements for dresser-drawer fronts over the planning period from week 4 through week 10:

Week	4	5	6	7	8	9	10
Gross Requirements	7500	0	14,500	0	0	13,500	10,000

The company has 2500 drawer fronts available at the end of week 3 and desires a safety stock of 100 units. Lead time for drawer fronts is one week, and the components are produced in lot sizes of 10,000 units. Determine the MRP planning worksheet for drawer fronts over the planning period from week 3 through week 10.

14. BFC (see problems 11 through 13) orders handles in lot sizes of 25,000. The lead time for handles is two weeks, and the company desires a safety stock of 4000 handles. If 6000 handles are available at the end of week 2, determine the MRP planning worksheet for handles over the period from week 1 through week 10. Base your gross requirements for handles on the answer you obtained in problem 13.

15. Frank's Garden Center forecasts demand for its products based on the previous year's sales. The demand for McMurray snow blowers over the same 10-week winter selling season last year was as follows.

Week	1	2	3	4	5	6	7	8	9	10
Demand	6	12	16	18	12	8	6	6	4	2

The store currently has one McMurray snow blower in stock and would like to have two left at the end of week 10. The cost of placing an order with McMurray for snow blowers is $50, and the cost of holding a snow blower in inventory is an estimated $1 per week.

a. Determine the inventory policy and inventory cost over the 10-week period if Frank's uses the EOQ formula for determining the order quantity. (Assume that annual demand is forecast to be 90 units.)
b. Determine the inventory policy and inventory cost over the 10-week period if Frank's uses the Silver-Meal heuristic for determining order quantities.
c. Determine the inventory policy and inventory cost over the 10-week period if Frank's uses the Wagner-Whitin algorithm for determining order quantities.

16. Whirling Blender manufactures electric blenders for sale to commercial customers. Whirling estimates the following customer demand over the eight-week planning cycle from week 10 through week 17.

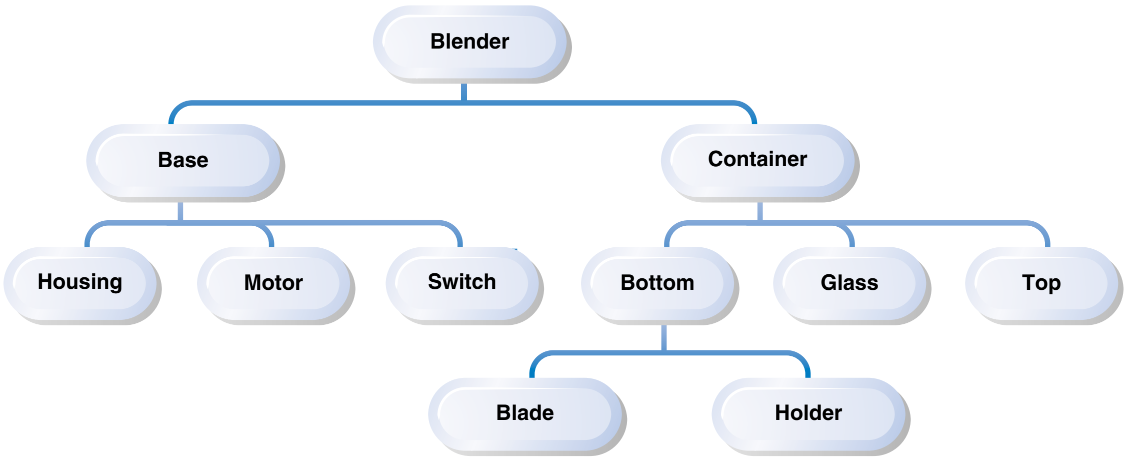
Week	10	11	12	13	14	15	16	17
Demand	1200	1400	800	1000	700	1500	1200	900

The product tree on the following page represents the bill of materials for the electric blender:

Whirling produces the blenders in batch sizes of 3000 units. Lead time for assembling the finished product is one week. Whirling desires a safety stock of 500 units and estimates that 800 blenders will be in inventory at the beginning of week 10.

a. Construct the MRP planning worksheet for blenders during the period from week 9 through week 17.
b. Whirling produces bases in lots of 2000 units and desires no safety stock. The lead time for base assembly is two weeks. If there is a balance of 500 bases in inventory at the beginning of week 9, determine the MRP planning worksheet for bases during the period from week 7 through week 17.
c. Whirling orders motors on a lot-for-lot basis. Motors have a lead time of three weeks. If the company desires a safety stock of 400 motors, and a balance of 600 motors is available at the beginning of week 7, determine the MRP planning worksheet for motors during the period from week 4 through week 17.

Product Tree for Problem 16



17. Consider the data given in problem 16 for Whirling blenders. The lead time and current inventory for each of the components are as follows.

Item	*Lead Time (in weeks)*	*Current Inventory*
Blender	1	300
Base	2	200
Housing	4	100
Motor	3	300
Switch	2	1000
Container	1	700
Bottom	2	200
Blade	5	500
Holder	3	100
Glass	6	400
Top	4	600

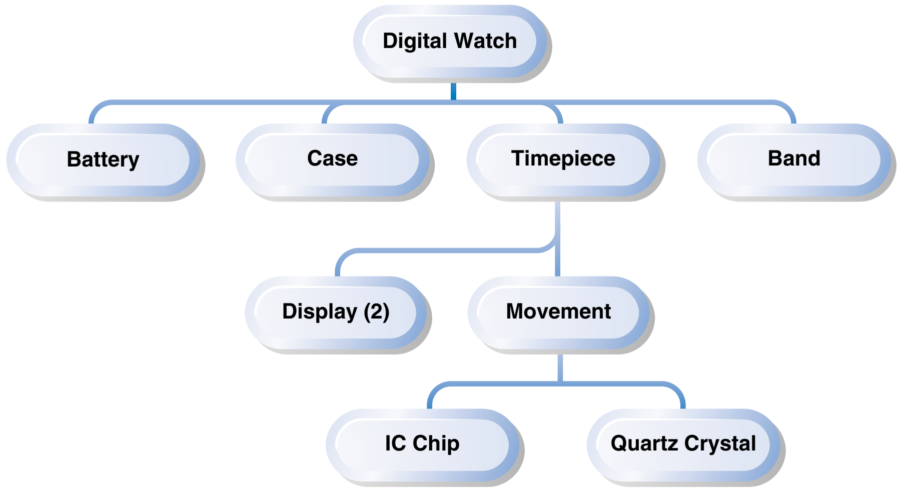
a. Determine the latest order date for each component in order to satisfy the demand in week 10.

b. Determine how much of each component should be ordered to satisfy the demand in week 10.

CASE 1: SWITCH Watch Company

SWITCH Watch Company manufactures watches with exchangeable fronts that customers can switch depending on their fashion mood. The company is planning its production of the men's 4320 Dual Display watch, which has two displays to enable the wearer to tell the time in two different parts of the world. The principal market for this watch is the business traveler; hence, it is heavily advertised in airline magazines.

The following product tree represents the bill of materials for this watch.



Safety stock requirements, lead times, and production lot sizes for the components are as follows.

Item	Safety Stock	Lead Time	Lot Size
Digital watches	200 units	1 week	500
Batteries	400 units	3 weeks	800
Cases	none	2 weeks	lot for lot
Timepieces	none	1 week	400
Displays	300 units	4 weeks	1200
Movements	100 units	2 weeks	lot for lot
IC chips	150 units	3 weeks	800
Quartz crystals	450 units	2 weeks	1400
Watch bands	500 units	5 weeks	2000

Demand for the watch over the planning period (week 8 through week 14) has been forecast as follows:

Week	8	9	10	11	12	13	14
Demand	350	500	400	650	800	400	300

Prepare a management report including MRP planning worksheets for each component over the relevant time period up to week 14. Discuss the ramifications of the watch's having two displays instead of a single display.