Selecting Multiple Projects

Based on the discussion presented in *Practical Management Science* (Third Edition) by Winston and Albright (2007).

Many companies have limited labor resources and multiple projects than can (or must) be completed. Selecting the projects to undertake is a very important problem for any company. The company must select a portfolio of projects that is consistent with its overall goals and strategy, provides desired diversification, maintains adequate cash flows, does not exceed resource availabilities, and does not exceed a reasonable level of risk.

The following example illustrates on possible model for project portfolio selection. In this model, we assume that each potential project has a worker requirement over some duration and a deadline. If the project is completed by the deadline, the company receives a reward; otherwise, it receives no reward. To simplify the example, we consider each project as a single activity (rather than as a series of activities).

EXAMPLE. Timburton Construction has 10 projects that it can (if desired) complete within the next 10 months. Each project earns a certain revenue when it is completed, but only if it is completed within the next 10 months. Otherwise the project earns no revenue. The number of workers needed each month, the number of months need to complete each project, and the revenue earned from each completed project are provided in the table below. We assume that after the company begins working on a project, it must work on the project during consecutive months until the project is completed. Timburton has 220 workers available each month. How can it maximize the revenue earned during the next 10 months?

Project	Workers per Month	Months	Late Start	Revenue
1	74	5	6	4800
2	98	2	9	3300
3	91	3	8	4100
4	95	4	7	6840
5	59	2	9	1650
6	81	3	8	3880
7	84	4	7	6380
8	78	3	8	4200
9	95	3	8	4860
10	58	5	6	5220

FORMULATION. Let

$$x_{ij} = \begin{cases} 1 & \text{if project } i \text{ starts in month } j \\ 0 & \text{o.w.} \end{cases}$$

$$y_{ij} = \begin{cases} 1 & \text{if project } i \text{ uses labor in month } j \\ 0 & \text{o.w.} \end{cases}$$

The x_{ij} determine whether or not the company decides to add project i to the portfolio during the next 10 months. The y_{ij} determine the months in which a selected project will be accomplished.

OBJECTIVE. We wish to maximize revenue which is given by

$$\sum_{i=1}^{10} \left(R_i \sum_{j=1}^{10} x_{ij} \right)$$

where R_i represents the revenue from project i if it is completed within the next 10 months.

CONSTRAINTS. There are 4 types of constraints in this model.

1. Bind the x and y variables.
   Let d_i be the duration of project i. A project can start in any month k as long as $1 \leq k$ and $k + d_i - 1 \leq 10$. For example, if project 1 starts in month $k = 4$ it is worked on for $d_i = 5$ months: $j = 4, 5, 6, 7, 8$ and $8 = k + d_i - 1 = 4 + 5 - 1$. But project 1 could not start in month $k = 7$ because $7 + 5 - 1 = 11 > 10$. It will be convenient to introduce the parameter k_i for each project i where $k_i + d_i - 1 = 10$. Thus, k_i is the latest month in which a project could start. (The late start for each project is provided in the data table.) The constraint is:

$$d_i x_{ij} \leq \sum_{j=k}^{k+d_i-1} y_{ij}, \quad k = 1, \dots k_i, \quad i = 1, \dots 10$$

2. Determine if project i will be added to the portfolio.
   This will also determine the start date of the project.

$$\sum_{j=1}^{k_i} x_{ij} \leq 1 \quad i = 1, \dots, 10$$

3. Labor Utilization.
   Let L_i be the monthly labor requirement for project i. Then each month's labor usage must not exceed the available labor:

$$\sum_{i=1}^{10} L_i y_{ij} \leq 220 \quad j = 1, \dots 10$$

4. All variables are binary.

$$x_{ij}, y_{ij} \in \{0, 1\} \ \forall \ i, j$$

OPTIMAL SOLUTION. This model has almost 200 variables (we can eliminate a few of the x_{ij}) and over 100 constraints. Although it is not too large to formulate in Excel, the software would likely require some time to find an optimal solution. The Excel formulation is provided in the workbook *MultProjSched.xls*. Winston & Albright use an Excel add-in called **Evolutionary Solver** (and a different formulation!) to obtain the solution: $x_{29} = x_{38} = x_{44} = x_{54} = x_{61} = x_{81} = x_{96} = x_{10,1} = 1$ and $x_{ij} = 0$ otherwise. However, the authors note that a better solution may be possible. More powerful software (e.g., LINDO, MATLAB) may be more successful in finding an optimal solution.

EXTENSIONS. Here are just a few possible extensions to this example.

- Each project could have its own due date. Maximize revenue.
- Each project could have its own due date. Maximize the number of projects completed on time.
- Monthly labor requirements for a project could vary.
- A project could have multiple activities that must be completed in a particular sequence before the project is considered finished.