·64.25)

∞)

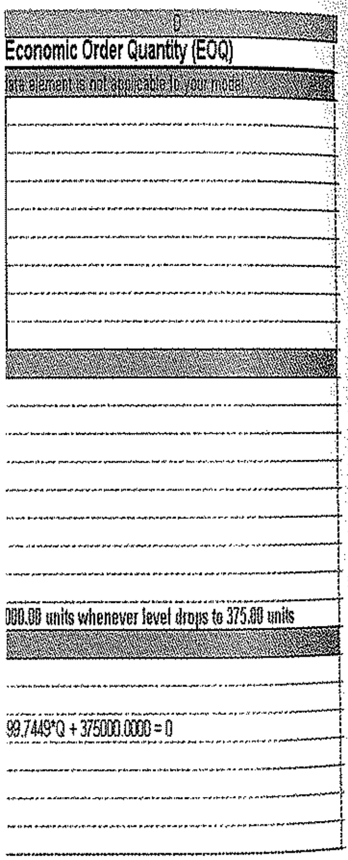
timal order quantity is $y^* =$

$2 \times 187.5 = 375$ gallons. Thus,

rops to 375 gallons

OQ.xls is designed to handle
m 9, Set 11.2a where uniform
es the price breaks situation

) the price breaks problem of
Enter the data of the model in
-1)-entry is required if a data


Economic Order Quantity (EOQ)

ata element is not applicable to your model

000.00 units whenever level drops to 375.00 units

99.7449*Q + 375000.0000 = 0

element does not apply to your model. For example, to solve EOQ models without price breaks, enter -1 for C_1, q, and c_2 (cells C3:C5). The system will display appropriate error messages to resolve input data conflicts. The output of the model gives the optimal inventory policy as well as all the intermediate calculations of the problem.

PROBLEM SET 11.2B

1. Consider the hotel laundry service situation in Problem 6, Set 11.2a. The normal charge for washing a soiled towel is \$.60, but the laundry service will charge only \$.50 if the hotel supplies them in lots of at least 2500 towels. Should the hotel take advantage of the discount?
2. An item is consumed at the rate of 30 items per day. The holding cost per unit per day is \$.05, and the setup cost is \$100. Suppose that no shortage is allowed and that the purchasing cost per unit is \$10 for any quantity not exceeding 500 units and \$8 otherwise.
   (a) Determine the optimal inventory policy, given a 21-day lead time.
   (b) Use the spreadsheet ch11EOQ.xls to solve the problem.
3. An item sells for \$25 a unit, but a 10% discount is offered for lots of 150 units or more. A company uses this item at the rate of 20 units per day. The setup cost for ordering a lot is \$50, and the holding cost per unit per day is \$.30. Should the company take advantage of the discount?
4. In Problem 3, determine the range on the price discount percentage that when offered for lots of size 150 units or more will not result in any financial advantage to the company.
5. In the inventory model discussed in this section, suppose that the holding cost per unit per unit time is h_1 for quantities below q and h_2 otherwise, $h_1 > h_2$. Show how the economic lot size is determined.

11.2.3 Multi-Item EOQ with Storage Limitation

This model deals with n (>1) items, whose individual inventory fluctuations follow the pattern in Figure 11.1 (no shortage allowed). The difference is that the items are competing for a limited storage space.

Define for item i, $i = 1, 2, \ldots, n$,

D_i = Demand rate
K_i = Setup cost
h_i = Unit holding cost per unit time
y_i = Order quantity
a_i = Storage area requirement per inventory unit
A = Maximum available storage area for all n items

Under the assumption of no shortage, the mathematical model representing the inventory situation is given as

$$\text{Minimize TCU}(y_1, y_2, \ldots, y_n) = \sum_{i=1}^{n}\left(\frac{K_iD_i}{y_i} + \frac{h_iy_i}{2}\right)$$

subject to

$$\sum_{i=1}^{n} a_i y_i \leq A$$

$$y_i > 0,\ i = 1, 2, \ldots, n$$

The steps for the solution of the model are

Step 1. Compute the *unconstrained* optimal values of the order quantities as

$$y_i^* = \sqrt{\frac{2K_iD_i}{h_i}},\ i = 1, 2, \ldots, n$$

Step 2. Check if the unconstrained optimal values y_i^* satisfy the storage constraint. If it does, stop; the solution y_i^*, $i = 1, 2, \ldots, n$, is optimal. Otherwise, go to step 3.

Step 3. The storage constraint must be satisfied in equation form. Use the Lagrange multipliers method to determine the constrained optimal values of the order quantities.

In step 3, the Lagrangean function is formulated as

$$\begin{aligned} L(\lambda, y_1, y_2, \ldots, y_n) &= \text{TCU}(y_1, y_2, \ldots, y_n) - \lambda\left(\sum_{i=1}^{n} a_i y_i - A\right) \\ &= \sum_{i=1}^{n}\left(\frac{K_iD_i}{y_i} + \frac{h_i y_i}{2}\right) - \lambda\left(\sum_{i=1}^{n} a_i y_i - A\right) \end{aligned}$$

where $\lambda(<0)$ is the Lagrange multiplier.[1]

Because the Lagrangean function is convex, the optimal values of y_i and λ are determined from the following necessary condition:

$$\frac{\partial L}{\partial y_i} = -\frac{K_iD_i}{y_i^2} + \frac{h_i}{2} - \lambda a_i = 0$$

$$\frac{\partial L}{\partial \lambda} = -\sum_{i=1}^{n} a_i y_i + A = 0$$

The second equation shows that the storage constraint must be satisfied in equation form at the optimum.

From the first equation,

$$y_i^* = \sqrt{\frac{2K_iD_i}{h_i - 2\lambda^* a_i}}$$

The formula shows that y_i^* is dependent on the value of λ^*. For $\lambda^* = 0$, y_i^* gives the unconstrained solution.

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[1] See Section 20.1.1 for the details of the Lagrangean method. The application of the method is correct in this case because $\text{TCU}(y_1, y_2, \ldots, y_n)$ is convex, and the problem has a single linear constraint and hence a convex solution space. The procedure may not be correct under other conditions or when the problem has more than one constraint as explained in Section 20.1.2.

The value of λ^* can be found in the following manner: Because by definition $\lambda < 0$ for the minimization case, we successively decrement λ by a reasonably small amount and use it in the given formula to compute the associated y_i^*. The desired λ^* yields the values of y_i^* that satisfy the storage constraint in equation form.

Example 11.2-3

The following data describe three inventory items.

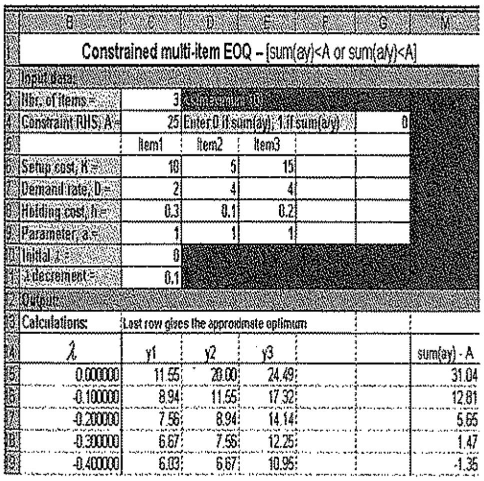
Item i	K_i (\$)	D_i (units per day)	h_i (\$)	$a_i(\text{ft}^2)$
1	10	2	.30	1
2	5	4	.10	1
3	15	4	.20	1
		Total available storage area $= 25\ \text{ft}^2$		

The computations associated with the model are simple yet tedious. The spreadsheet template ch11ConstrainedEOQ.xls is provided to alleviate this difficulty.

Figure 11.6 shows the application of the template to the data of this example. The input data involves all the necessary parameters for all the items. The initial value of λ (Initial Lambda) is usually set equal to zero, and the decrement in λ (Lambda decrement) is set to a reasonable value. These initial values can be adjusted to secure any degree of accuracy in the calculations, as we will explain shortly. The template can handle a maximum of 10 items. The template is also designed to account for problems in which the constraint takes the form

$$\sum_{i=1}^{n} \frac{a_i}{y_i} \leq A$$

This type of constraint may arise in other situations as demonstrated by Problem 4, Set 11.2c. To use this option, you must enter 1 in cell G4 of the template.



	B	C	D	E	F	G	M
1	Constrained multi-item EOQ – [sum(ay)<A or sum(a/y)<A]						
2	Input data:						
3	Nbr. of items =	3	[illegible]				
4	Constraint RHS, A =	25	Enter 0 if sum(ay), 1 if sum(a/y)			0	
5		Item1	Item2	Item3			
6	Setup cost, K =	10	5	15			
7	Demand rate, D =	2	4	4			
8	Holding cost, h =	0.3	0.1	0.2			
9	Parameter, a =	1	1	1			
10	Initial λ =	0					
11	λ decrement =	0.1					
12	Output:						
13	Calculations:	Last row gives the approximate optimum					
14	λ	y1	y2	y3			sum(ay) - A
15	0.000000	11.55	20.00	24.49			31.04
16	-0.100000	8.94	11.55	17.32			12.81
17	-0.200000	7.56	8.94	14.14			5.65
18	-0.300000	6.67	7.56	12.25			1.47
19	-0.400000	6.03	6.67	10.95			-1.35

FIGURE 11.6

Excel solution of the storage model of Example 11.2-3

The last column in Figure 11.6 shows that the storage equation is satisfied somewhere in the range $-.3 > \lambda > -.4$. The Excel spreadsheet can refine the answer to any desired degree of accuracy. Here is how: Change the value of Initial Lambda (cell C10) to $-.3$ and specify a smaller Lambda decrement (cell C11), say .05. Now, execute the model and revise Initial Lambda and decrement again. Repeat the process, always updating Initial Lambda and selecting smaller Lambda decrement until the desired accuracy is achieved. I tried this procedure and was able to refine the search to Initial Lambda $= -.03475$ and Lambda decrement $= .0005$. At $\lambda^* = -.348$, the equation netted to almost zero. The corresponding values of the order quantities are given as

$$y_1^* \approx 6.34 \text{ units},\ y_2^* \approx 7.09 \text{ units},\ y_3^* \approx 11.57 \text{ units}$$

PROBLEM SET 11.2C[2]

1. The following data describe five inventory items.

Item i	K_i (\$)	D_i (units per day)	h_i (\$)	a_i(ft^2)
1	20	22	0.35	1.0
2	25	34	0.15	0.8
3	30	14	0.28	1.1
4	28	21	0.30	0.5
5	35	26	0.42	1.2

Total available storage area = 25 ft^2

Determine the optimal order quantities.

2. Solve the model of Example 11.2-3, assuming that we require the sum of the average inventories for all the items to be less than 25 units.
3. In Problem 2, assume that the only restriction is a limit of \$1000 on the amount of capital that can be invested in inventory. The purchase costs per unit of items 1, 2, and 3 are \$100, \$50, and \$100, respectively. Determine the optimum solution.
4. The following data describe four inventory items.

Item i	K_i (\$)	D_i (units per day)	h_i (\$)
1	100	10	.1
2	50	20	.2
3	90	5	.2
4	20	10	.1

The company wishes to determine the economic order quantity for each of the four items such that the total number of orders per year (365 days) is at most 150. Set up the Lagrangean function and develop the necessary formulas; then use ch11ConstrainedEOQ.xls to solve the problem.

5. Use the partial derivative equations of the inventory model in this section to show that the starting value of optimal λ can be approximated by

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[2] You will find ch11StorageEOQ.xls useful in solving the problems of this set.

11.3 DYN

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FIGURE 1 Example

$$\lambda^* \approx \frac{\overline{h}}{2\overline{a}} - \frac{n^2\overline{a}\,\overline{KD}}{A^2}$$

where

$$\overline{h} = \frac{\sum_{i=1}^{n} h_i}{n},\ \overline{a} = \frac{\sum_{i=1}^{n} a_i}{n},\ \overline{KD} = \frac{\sum_{i=1}^{n} K_iD_i}{n}$$

11.3 DYNAMIC EOQ MODELS

The models presented here differ from those in Section 11.2 in two aspects: (1) The inventory level is reviewed periodically over a finite number of equal periods; and (2) the demand per period, though deterministic, is dynamic, in the sense that it may vary from one period to the next.

A situation in which dynamic deterministic demand occurs is **materials requirement planning** (MRP). The idea of MRP is described by an example. Suppose that the quarterly demand over the next year for two final models, *M*1 and *M*2, of a given product is 100 and 150 units, respectively. Deliveries of the quarterly lots are made at the end of each quarter. The production lead time is 2 months for *M*1 and 1 month for *M*2. Each unit of *M*1 and *M*2 uses 2 units of a subassembly *S*. The lead time for the production of *S* is 1 month.

Figure 11.7 depicts the production schedules for *M*1 and *M*2. The schedules start with the quarterly demand for the two models (shown by solid arrows) occurring at the

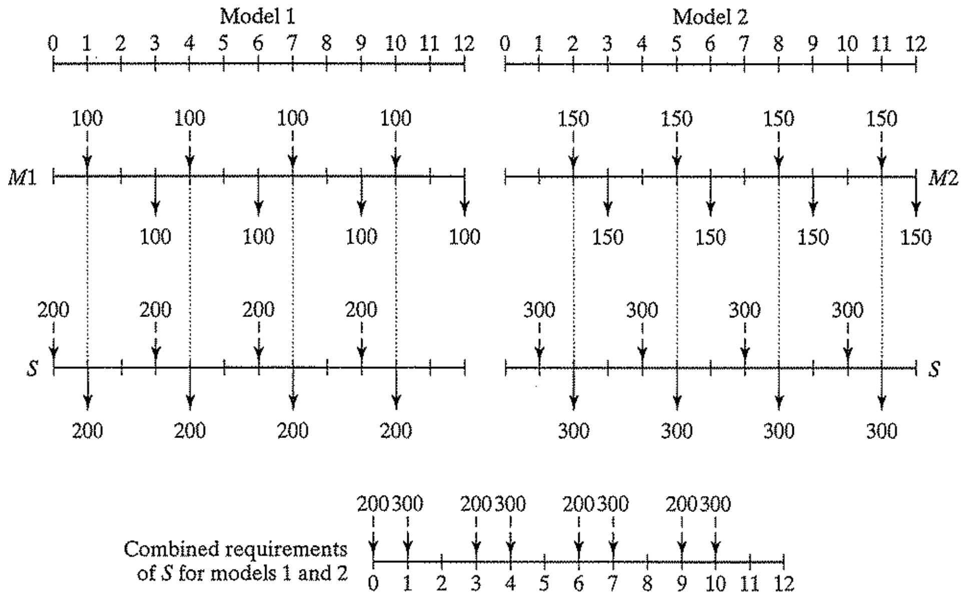

FIGURE 11.7

Example of dynamic demand generated by MRP