

Linear Programming Example: Feed Mix

Feed mix contains two active ingredients and a filler to provide bulk. The table below shows the amount of four different nutrients contained in each ingredient, the requirement for each nutrient, and the unit cost for each ingredient.

<i>Ingredient</i>	<i>Nutrient A</i> (g/kg)	<i>Nutrient D</i> (g/kg)	<i>Nutrient C</i> (g/kg)	<i>Nutrient D</i> (g/kg)	<i>Unit Cost</i> (\$/kg)
1	100	80	40	10	4
2	200	150	20	0	6
<i>Required</i> (g/kg)	90	50	20	2	

For example, one kilogram of Ingredient 1 contains 100g Nutrient A. Formulate and solve a linear program to determine how much of each ingredient and how much filler should be used to make one kilogram of feed mix at minimum cost while meeting the nutritional requirements.

ANSWER:

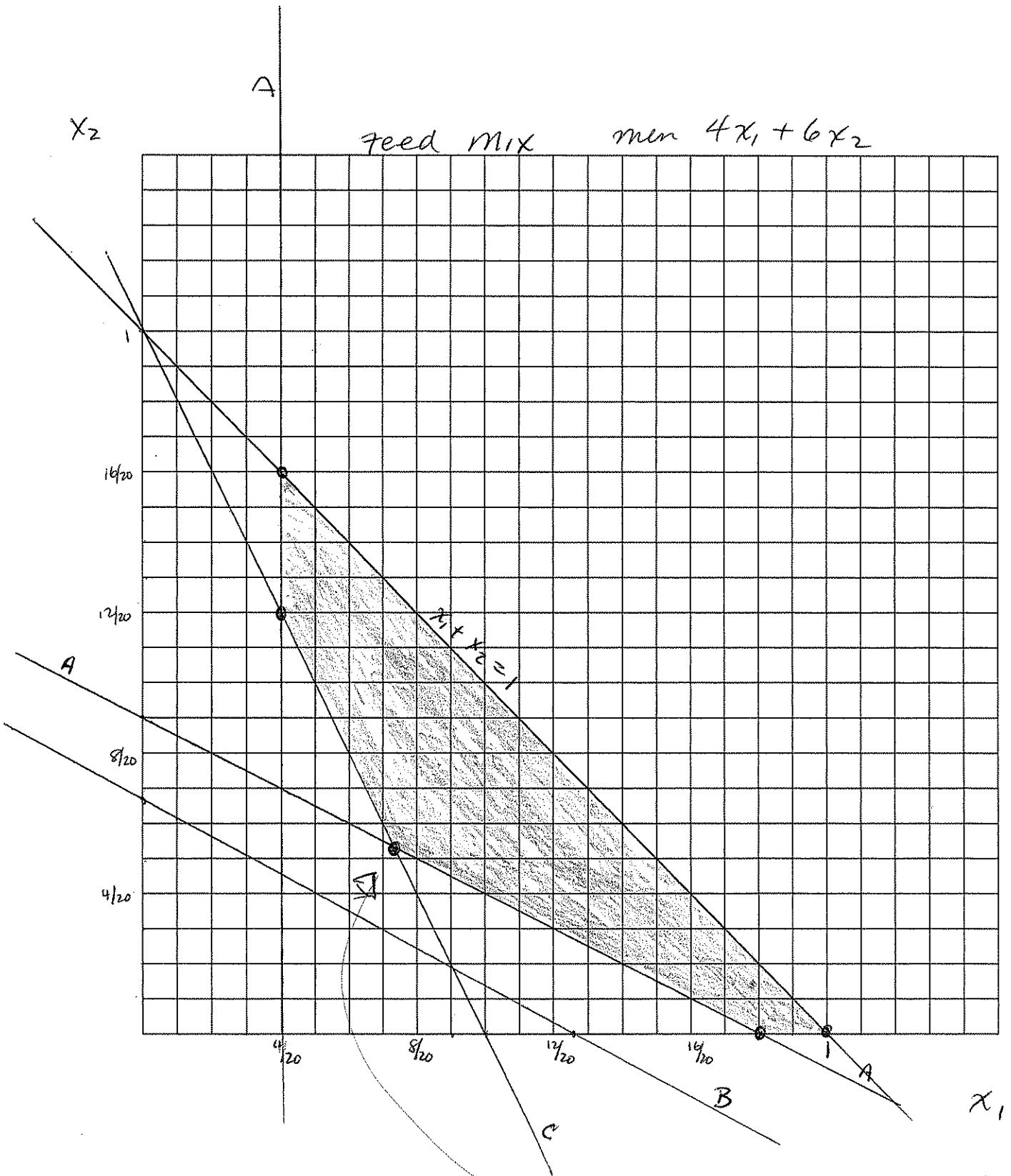
Let x_1 = amount (kg) of Ingredient 1 used in one kilogram of feed mix
 x_2 = amount (kg) of Ingredient 2 used in one kilogram of feed mix
 x_3 = amount (kg) of filler used in one kilogram of feed mix

$$\begin{array}{llllllll}
 \text{min} & 4x_1 & + & 6x_2 & & & & \text{(cost)} \\
 \text{s.t.} & 100x_1 & + & 200x_2 & & \geq & 90 & \text{(A)} \\
 & 80x_1 & + & 150x_2 & & \geq & 50 & \text{(B)} \\
 & 40x_1 & + & 20x_2 & & \geq & 20 & \text{(C)} \\
 & 10x_1 & & & & \geq & 2 & \text{(D)} \\
 & x_1 & + & x_2 & + & x_3 & = & 1 \quad \text{(feed mix)} \\
 & x_1 & , & x_2 & , & x_3 & \geq & 0 \quad \text{(nonneg)}
 \end{array}$$

We can solve the problem graphically by eliminating x_3 from the formulation. Using the feed mix EQUATION, $x_3 = 1 - x_1 - x_2$. Since $x_3 \geq 0$, $1 - x_1 - x_2 \geq 0$ or $x_1 + x_2 \leq 1$. The two-variable formulation is:

min	$4x_1$	+	$6x_2$			(cost)
s.t.	$100x_1$	+	$200x_2$	\geq	90	(A)
	$80x_1$	+	$150x_2$	\geq	50	(B)
	$40x_1$	+	$20x_2$	\geq	20	(C)
	$10x_1$			\geq	2	(D)
	x_1	+	x_2	\leq	1	(feed mix)
	x_1	,	x_2	\geq	0	(nonneg)

The graph is shown below. Alternatively, we can solve the model using Excel. The spreadsheet is available online.



nutrient B constraint is redundant

$$\begin{aligned} x_1^* &= 11/30 \\ x_2^* &= 8/30 \\ x_3^* &= 11/30 \end{aligned}$$