BAJA BURRITO RESTAURANTS

Baja Burrito Restaurants, commonly called BB's, is a chain of fast-food Mexican-style restaurants with over 100 locations throughout the Southwest and Great Plains states. It features such items as the Taco Loco Grande, the Quesadilla Quatro (a quesadilla made with four different cheeses), and specialty burritos named after many of the states BB's services (such as the Arizona burrito, the Texas burrito, and the Oklahoma burrito). Like most fast-food restaurants, the basic style and décor of the individual restaurants do not vary greatly from restaurant to restaurant, although the basic design must be modified somewhat to adjust to size constraints and to conform to local codes and ordinances.

BB's is planning to open a new restaurant in Lubbock, Texas, near the site where Texas Tech University plays its football games. It wishes to have the restaurant built and operational prior to the first Texas Tech home football game on September 11, which is 19 weeks away. Table 5.12 details the activities and the immediate predecessor relations in the first two columns and the approximate normal times (in weeks) and costs (in $1000s) for building a restaurant in the third and fourth columns. Thus, under normal conditions, it costs BB's about $200,000 to construct a new restaurant.

Based on the normal time estimates, using the PERT-CPM.xls template, we can show that it will take 29 weeks for BB's to complete the construction project. Since this does not meet the 19-week deadline, management at BB's has requested department heads and project engineers look for ways to speed up the individual activities. These are reflected in the last two columns in Table 5.12. Therefore, if all activities are crashed to their minimum times, the restaurant could be built in 17 weeks at a cost of $300,000.

Management is willing to assume that spending additional funds up to the crash amounts submitted by the department heads and engineers will reduce the activity completion times proportionately. They are seeking a minimum cost activity schedule for building the new Lubbock Baja Burrito restaurant that will meet the 19-week deadline.

TABLE 5.12 Activity Chart for Baja Burrito Restaurants

Activity	Immediate Predecessors	Normal Time	Normal Cost	Crash Time	Crash Cost
A. Plan revisions/approvals	—	5	25	3	36
B. Grade land	A	1	10	0.5	15
C. Purchase materials	A	3	18	1.5	22
D. Order/receive equipment	A	2	8	1	12
E. Order/receive furniture	A	4	8	1.5	15
F. Pour concrete floor	B,C	1	12	0.5	15
G. Erect frame	F	4	20	2.5	30
H. Install electrical	G	2	12	1.5	17
I. Install plumbing	G	4	13	2.5	21
J. Install drywall/roof	H,I	2	10	1.5	16
K. Construct bathrooms	I	2	8	1	12
L. Install equipment	D,J	3	14	1.5	22
M. Finish/paint inside	K,L	3	10	1.5	18
N. Tile floors	M	3	6	1	9
O. Install furniture	E,M	4	8	2.5	14
P. Finish/paint outside	J	4	18	2.5	26
TOTAL		29*	$200	17*	$300

*As determined by the PERT.xls template.

SOLUTION

Figure 5.21 shows the PERT/CPM network for this model. If this model were solved by hand, the first step would be to determine whether construction of the restaurant would meet the 19-week deadline using the normal time/cost data. As mentioned earlier, using normal times/costs would give a 29-week completion time determined by a critical path consisting of activities A, C, F, G, I, J, L, M, and O.

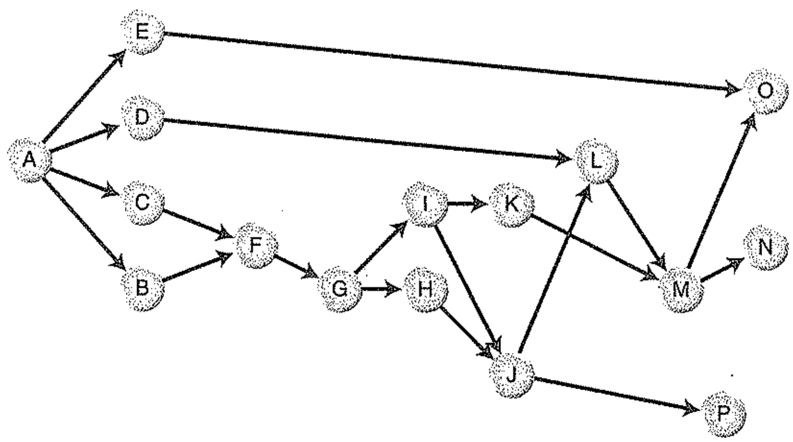


FIGURE 5.21
PERT/CPM Network for Baja Burrito Restaurants

Thus to meet the 19-week deadline, some of the activities must be crashed. Table 5.13 details the maximum time reductions, R $(=T_N - T_C)$, the extra costs for these reductions, E, and the marginal cost per week reductions, M (=E/R).

In order to reduce the project time, the completion time of one or more of the critical activities must be crashed. When the completion time for a critical activity is reduced by a large enough amount, however, other paths will also become critical. To achieve further time reductions, activities on *all* critical paths must be crashed.

TABLE 5.13 R, E, and M Values for Baja Burrito

Activity	Maximum Reduction R	Extra Cost E	Cost Per Week Reduction M = E/R
A	2.0	11	5.50
B	0.5	5	10.00
C	1.5	4	2.67
D	1.0	4	4.00
E	2.5	7	2.80
F	0.5	3	6.00
G	1.5	10	6.67
H	0.5	5	10.00
I	1.5	8	5.33
J	0.5	6	12.00
K	1.0	4	4.00
L	1.5	8	5.33
M	1.5	8	5.33
N	2.0	3	1.50
O	1.5	6	4.00
P	1.5	8	5.33

A heuristic approach to determine the amount of time each activity should be crashed can be developed by taking into account the following: (1) the project time is reduced only when activities on the critical path are reduced; (2) the maximum time reduction for each activity is limited; and (3) the amount of time an activity on the critical path can be reduced before another path also becomes a critical path is limited. For *very* small problems, an approach based on these observations can work rather well, but as the number of critical paths increases, the procedure becomes cumbersome rather rapidly.

LINEAR PROGRAMMING APPROACH TO CRASHING

Fortunately, the use of such a heuristic approach is unnecessary. A simple modification to the linear program given in Section 5.4 is all that is required. For this model, we now define two variables for each activity, j.

X_j = start time for the activity
Y_j = the amount by which the activity is to be crashed

Since the normal cost must always be paid, the objective is to minimize the sum of the *additional* funds spent to reduce the completion times of activities. The cost per unit reduction for an activity is M_j, and the amount of time the activity is reduced is the decision variable, Y_j. Therefore, the total extra amount spent crashing the activity is M_jY_j. Because we want to minimize the total additional funds spent to crash the project, the objective function is the sum of all such costs:

$$\text{MIN} \sum_j M_jY_j$$

For Baja Burrito Restaurants the objective function is:

$$\begin{aligned}\text{MIN } & 5.5Y_A + 10Y_B + 2.67Y_C + 4Y_D + 2.8Y_E + 6Y_F + 6.67Y_G + 10Y_H + \\ & 5.33Y_I + 12Y_J + 4Y_K + 5.33Y_L + 5.33Y_M + 1.5Y_N + 4Y_O + 5.33Y_P\end{aligned}$$

Constraints

There are three types of constraints in this approach:

1. ***No activity can be reduced more than its maximum time reduction.***

For each activity, there is a constraint of the form:

$$Y_j \le R_j$$

2. ***The start time for an activity must be at least as great as the finish time of all immediate predecessor activities.***

This represents a series of constraints similar to those described in Section 5.4, of the form:

(Start Time for an Activity) ≥
(Finish Time for an Immediate Predecessor of the Activity)

Now, however, since the activity finish times are reduced by the amount of time each activity is crashed, these constraints have the form:

(Start Time for an Activity) ≥
(Start Time for a Predecessor Activity) +
(Normal Completion Time of the Predecessor Activity) −
(Time the Predecessor Activity is Crashed)

There is one such constraint for each immediate predecessor relationship. (This is equivalent to saying that there is one constraint for each arc in the PERT/CPM network.) In this project, for example, activity I (which has a normal completion time of four weeks) is one of the immediate predecessors for activity J. Thus one of the constraints in the linear programming formulation would be:

$$X_J \geq X_I + (4 - Y_I)$$

3. ***The project must be completed by its deadline, D.***
Since the project completion time is determined by the maximum of the finish times of the terminal activities[3] in the project (the ones that are not predecessors for any other activities), we add constraints of the form:

$$\text{(Finish Time for a Terminal Activity)} \leq D$$

or for each *terminal* activity,

$$\text{(Activity Start Time)} + \text{(Activity's Normal Completion Time)} - \text{(Time Activity is Crashed)} \leq D$$

In this model, activities N, O, and P, having normal completion times of 3, 4, and 4, respectively, are not predecessors for any other activities. Thus the following constraints would be added:

$$\begin{aligned} X_N + 3 - Y_N &\leq 19 \\ X_O + 4 - Y_O &\leq 19 \\ X_P + 4 - Y_P &\leq 19 \end{aligned}$$

The complete linear programming model for Baja Burrito Restaurants is then:

$$\begin{aligned} \text{MIN } & 5.5Y_A + 10Y_B + 2.67Y_C + 4Y_D + 2.8Y_E + 6Y_F + 6.67Y_G + 10Y_H \\ & + 5.33Y_I + 12Y_J + 4Y_K + 5.33Y_L + 5.33Y_M + 1.5Y_N + 4Y_O + 5.33Y_P \end{aligned}$$

ST

$$\left.\begin{array}{r} Y_A \leq 2.0 \\ Y_B \leq 0.5 \\ Y_C \leq 1.5 \\ Y_D \leq 1.0 \\ Y_E \leq 2.5 \\ Y_F \leq 0.5 \\ Y_G \leq 1.5 \\ Y_H \leq 0.5 \\ Y_I \leq 1.5 \\ Y_J \leq 0.5 \\ Y_K \leq 1.0 \\ Y_L \leq 1.5 \\ Y_M \leq 1.5 \\ Y_N \leq 2.0 \\ Y_O \leq 1.5 \\ Y_P \leq 1.5 \end{array}\right\} \quad (1)$$

[3] If you do not wish to identify the terminal activities, you can simply require *all* finish times not to exceed D. This will add several redundant constraints, but for small problems these will not significantly impact the solution time of the linear program.

$$\left.\begin{aligned}
X_B &\geq X_A + (5 - Y_A)\\
X_C &\geq X_A + (5 - Y_A)\\
X_D &\geq X_A + (5 - Y_A)\\
X_E &\geq X_A + (5 - Y_A)\\
X_F &\geq X_B + (1 - Y_B)\\
X_F &\geq X_C + (3 - Y_C)\\
X_G &\geq X_F + (1 - Y_F)\\
X_H &\geq X_G + (4 - Y_G)\\
X_I &\geq X_G + (4 - Y_G)\\
X_J &\geq X_H + (2 - Y_H)\\
X_J &\geq X_I + (4 - Y_I)\\
X_K &\geq X_I + (4 - Y_I)\\
X_L &\geq X_D + (2 - Y_D)\\
X_L &\geq X_J + (2 - Y_J)\\
X_M &\geq X_K + (2 - Y_K)\\
X_M &\geq X_L + (3 - Y_L)\\
X_N &\geq X_M + (3 - Y_M)\\
X_O &\geq X_E + (4 - Y_E)\\
X_O &\geq X_M + (3 - Y_M)\\
X_P &\geq X_J + (2 - Y_J)
\end{aligned}\right\} \quad (2)$$

$$\left.\begin{aligned}
X_N + 3 - Y_N &\leq 19\\
X_O + 4 - Y_O &\leq 19\\
X_P + 4 - Y_P &\leq 19
\end{aligned}\right\} \quad (3)$$

All X's and Y's ≥ 0

The constraints could be rewritten so that they resemble our usual linear programming form of having the variables on the left side of the constraints and only constants on the right. Regardless, we see that even for this relatively small problem, the linear program consists of 32 variables and 39 functional constraints.

USING CPM-DEADLINE.xls TEMPLATE

Fortunately, one does not actually have to write the linear program for CPM; many computer packages do this automatically. We have included the CPM-Deadline.xls template on the accompanying CD-ROM to do just that. Figure 5.22 shows the results from the CPM DEADLINE OUTPUT worksheet for the Baja Burrito Restaurant model. We see that the project can be completed in 19 weeks at a cost of $248,750 by crashing activities A, C, F, I, L, M, N, and O by certain amounts. A scheduling of the activities that meets this deadline is shown in columns C and D. (*Incidentally, the* 7.87637×10^{-11} *entry for the cost of crashing for activity G is simply a roundoff error generated internally from solving the linear program. This number is actually 0.*)

OPERATING OPTIMALLY WITHIN A FIXED BUDGET

The CPM approach presented for the Baja Burrito Restaurants model sought to find the minimum cost of constructing the restaurant within 19 weeks. Many projects, however, including construction projects, marketing campaigns, and research and development studies, must operate within a given fixed budget. In such cases, the objective is to complete the project in minimum time, subject to the budget restrictions. The CPM approach can be modified for these models.

Baja Deadline.xls

CRASHING ANALYSIS

TOTAL PROJECT COST	248.75	PROJECT NORMAL COST	200
COMPLETION TIME	19	PROJECT CRASH COST	300

ACTIVITY	NODE	Completion Time	Start Time	Finish Time	Amount Crashed	Cost of Crashing	Total Cost
Revisions/Approvals	A	3	0	3	2	11	36
Grade Land	B	1	3	4	0	0	10
Purchase Materials	C	1.5	3	4.5	1.5	4	22
Order Equipment	D	2	3	5	0	0	8
Order Furniture	E	4	12.5	16.5	0	0	8
Concrete Floor	F	0.5	4.5	5	0.5	3	15
Erect Frame	G	4	5	9	0	7.87637E-11	20
Install Electrical	H	2	9	11	0	0	12
Install Plumbing	I	2.5	9	11.5	1.5	8	21
Install Drywall/Roof	J	2	11.5	13.5	0	0	10
Bathrooms	K	2	13	15	0	0	8
Install Equipment	L	1.5	13.5	15	1.5	8	22
Finish/Paint Inside	M	1.5	15	16.5	1.5	8	18
Tile Floors	N	2.5	16.5	19	0.5	0.75	6.75
Install Furniture	O	2.5	16.5	19	1.5	6	14
Finish/Paint Outside	P	4	13.5	17.5	0	0	18

CPM DEADLINE INPUT / CPM DEADLINE OUTPUT

FIGURE 5.22
CPM DEADLINE OUTPUT
Worksheet for
Baja Restaurants

BAJA BURRITO RESTAURANTS (CONTINUED)

Baja Budget.xls

Suppose Baja Burrito has a policy of not funding a project for more than $12\frac{1}{2}\%$ above "normal cost" forecasts. In this case, $12\frac{1}{2}\% = \$25{,}000$, meaning that the maximum spending limit for the project would be $225,000. Given this spending limit, management is interested in the earliest it can expect the Lubbock BB's construction project to be completed.

SOLUTION

The problem is basically the same as that for the previous model, with the following exceptions:

1. The constraints, labeled (3), which state that the project must be completed within 19 weeks, are eliminated.
2. A new constraint is added stating that the maximum "extra spending" cannot exceed $25,000:

$$5.5Y_A + 10Y_B + 2.67Y_C + 4Y_D + 2.8Y_E + 6Y_F + 6.67Y_G + 10Y_H$$
$$+ 5.33Y_I + 12Y_J + 4Y_K + 5.33Y_L + 5.33Y_M + 1.5Y_N + 4Y_O + 5.33Y_P \leq 25$$

3. The objective is changed to Min(Max(EF)).

This objective function, however, is not a linear function. In addition, imposing a constraint similar to MIN ΣX's that we used in Section 5.4, this time does not guarantee an optimal solution.

To convert this to a linear program, we can imagine creating a dummy node called "END," signifying the end of the project. All termination activities, activities that are not predecessors for other activities, are then immediate predecessors of this END node. In this case, activities N, O, and P are not predecessors for any other activities and thus would be predecessors for this END node. The end of the PERT/CPM network for this model is shown in Figure 5.23.

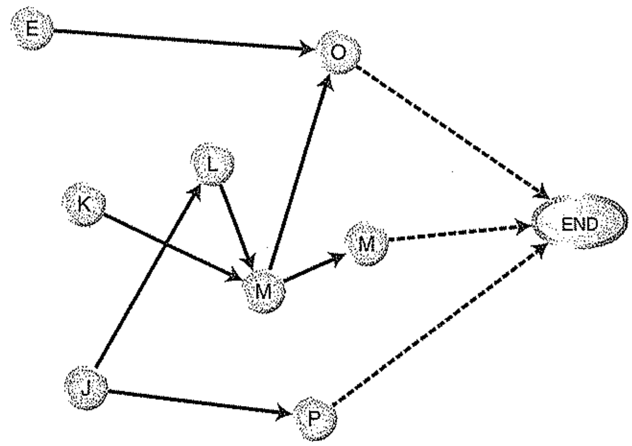


FIGURE 5.23
Termination Nodes (N, O, P) of the Baja Restaurant Network

The constraints of the previous model would then be amended by deleting tl constraints (3), adding the constraint above restricting extra spending to at mo $25,000, and adding three more precedence relation constraints:

$$X_{END} \geq X_N + (3 - Y_N)$$
$$X_{END} \geq X_O + (4 - Y_O)$$
$$X_{END} \geq X_P + (4 - Y_P)$$

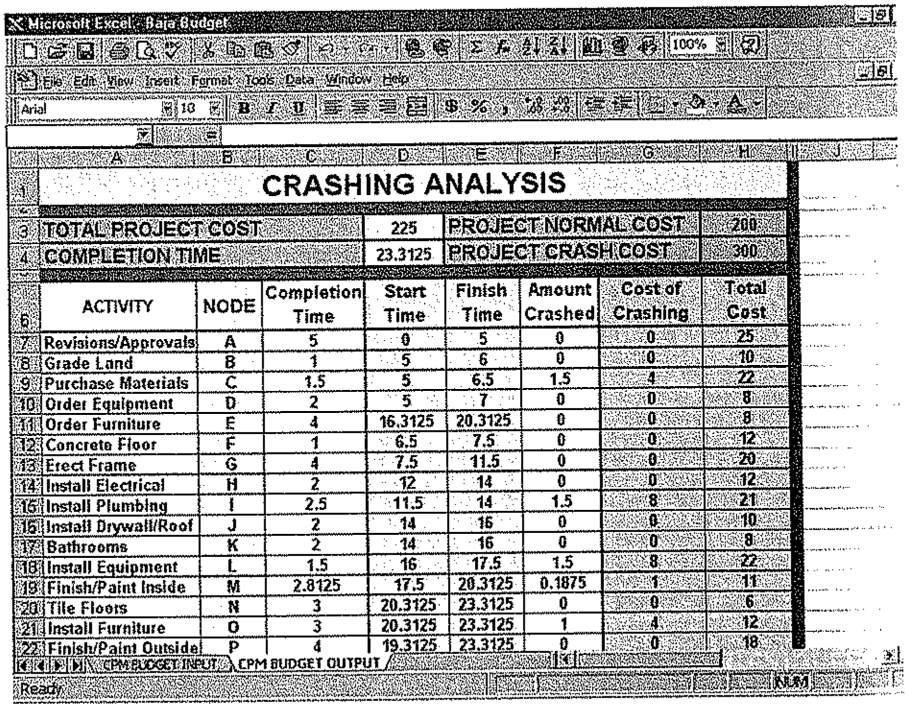
The *linear* objective now is:

$$\text{MIN } X_{END}$$

USING THE CPM-BUDGET.xls TEMPLATE

The above is precisely the approach used in the CPM-Budget.xls template. Tl template is designed specifically to solve project scheduling models with limit budgets. Input instructions are given in Appendix 5.1. Figure 5.24 shows the CP. BUDGET OUTPUT worksheet for the Baja Burrito model.

Baja Budget.xls



CRASHING ANALYSIS

TOTAL PROJECT COST	225	PROJECT NORMAL COST	200
COMPLETION TIME	23.3125	PROJECT CRASH COST	300

ACTIVITY	NODE	Completion Time	Start Time	Finish Time	Amount Crashed	Cost of Crashing	Total Cost
Revisions/Approvals	A	5	0	5	0	0	25
Grade Land	B	1	5	6	0	0	10
Purchase Materials	C	1.5	5	6.5	1.5	4	22
Order Equipment	D	2	5	7	0	0	8
Order Furniture	E	4	16.3125	20.3125	0	0	8
Concrete Floor	F	1	6.5	7.5	0	0	12
Erect Frame	G	4	7.5	11.5	0	0	20
Install Electrical	H	2	12	14	0	0	12
Install Plumbing	I	2.5	11.5	14	1.5	8	21
Install Drywall/Roof	J	2	14	16	0	0	10
Bathrooms	K	2	14	16	0	0	8
Install Equipment	L	1.5	16	17.5	1.5	8	22
Finish/Paint Inside	M	2.8125	17.5	20.3125	0.1875	1	11
Tile Floors	N	3	20.3125	23.3125	0	0	6
Install Furniture	O	3	20.3125	23.3125	1	4	12
Finish/Paint Outside	P	4	19.3125	23.3125	0	0	18

FIGURE 5.24
CPM BUDGET OUTPUT Worksheet for Baja Burrito Restaurants