Practice Final  
MAT-16C  Short Calculus - III  
Spring 2011  

Name

- This test is closed notes, closed book.
- Laptops and calculators are NOT allowed.
- There are 4 pages and 11 questions total.
- You can leave an answer as a numerical expression without computing the final value. For example, this is a perfectly acceptable answer: 
  \[ \frac{((250 - 63)/(1 - e^{-6.5})) * \ln(27/168)}{27/168} \]. Show your work clearly!!
- The maximum score in the test is 150 points.

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1. Let \( f(x, y) = \cos(1 + xy^2) \).
   
   (a) (4 pts) Compute \( f_y \).
   (b) (6 pts) Compute \( f_{yx} \).

2. Consider the function \( f(x, y) = 5 + x^2 - x^2y - y^2 - \frac{1}{3}y^3 \).
   
   (a) (8 pts) Find all critical points of \( f(x, y) \).
   (b) (7 pts) Decide whether each critical point found in (a) is a relative minimum, relative maximum, saddle point or indeterminable.

3. Write the \( n \)-th term in the following sequences:
   
   (a) (2 pts) \(-\frac{1}{2}, \frac{4}{3}, -\frac{9}{4}, \frac{16}{5}, \ldots \)
   (b) (2 pts) \(2, 2, \frac{8}{6}, \frac{16}{24}, \frac{32}{120}, \ldots \)
   (c) (2 pts) \(1, \frac{5}{8}, \frac{7}{15}, \frac{9}{24}, \frac{11}{35}, \frac{13}{48}, \ldots \)

4. Determine if the following sequences converge or diverge. If the sequence converges, write the limit.
   
   (a) (3 pts) \( a_n = \frac{2^{n+1}}{3\cdot 2^n} \)
   (b) (3 pts) \( a_n = (-1)^n \left( \frac{1}{n^2+3} \right) \)
   (c) (3 pts) \( a_n = (-1)^n \left( \frac{n+1}{n} \right) \)

5. Determine if the following series converge or diverge. Clearly explain why.
   
   (a) (6 pts) \( \sum_{n=0}^{\infty} \frac{n}{500n+79} \)
   (b) (6 pts) \( \sum_{n=1}^{\infty} (2n)! \left( \frac{2}{3} \right)^n \)
   (c) (6 pts) \( \sum_{n=1}^{\infty} \frac{1}{n^{\sqrt{n}}} \)

6. (12 pts) Find the radius and interval of convergence for the power series
   \[ \sum_{n=1}^{\infty} \frac{3^{-n}}{n+1} (x + 1)^n. \]

7. (10 pts) Find the sum \( \sum_{n=0}^{\infty} \frac{1}{3^n 2^{n-2}} \).

8. Evaluate the following double integrals. (Remember that sometimes it helps to change the order of integration).

   (a) (12 pts) \( \int_0^2 \int_0^{\sqrt{x}} y(x - y^2)^3 \, dy \, dx \).
   (b) (12 pts) \( \int_0^4 \int_{\sqrt{x}}^2 xsin(1 + y^5) \, dy \, dx \).
9. Solve the following differential equations.

(a) (10 pts) \( e^{x^2}x' + y = 2xy. \)

(b) (10 pts) \( xy' - 2y = x^2\ln(x) \) with the initial conditions that \( y = 0 \) when \( x = 1. \)

10. (15 pts) Approximate the definite integral \( \int_0^1 xe^{-x^3} dx \) using a 7th-degree Taylor polynomial for the function \( xe^{-x^3}. \)

11. (11 pts) Minimize the function \( f(x, y, z) = x^2 + 2y^2 + 3z^2 \) subject to the constraint \( 3x - 2y + z = \frac{34}{6}. \) Find \( x, y, z \) at the minimum and the minimum value of the function.