Practice Final
MAT-16B   Short Calculus - II
Spring 2011

Name ____________________________________________

• This test is closed notes, closed book.
• Calculators ARE allowed in this final.
• There are 6 pages and 10 questions total.
• For numerical answers, compute the final value using your calculator. **DO NOT** leave it as a long expression like:

\[
\frac{(250 - 63)}{(1 - e^{(-6.5)})} \times \ln(27/168).
\]

Show your work clearly !!

• The maximum score in the test is 150 points.

Signature ____________________________________________
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1. Compute the following integrals (note that some of them are indefinite integrals and some are definite integrals).

(a) **(6 pts)** \( \int \frac{e^{7x}}{50+e^{7x}} \, dx \).

(b) **(7 pts)** \( \int x\sqrt{5-x} \, dx \).

(c) **(7 pts)** \( \int x\ln(x+1) \, dx \).

(d) **(7 pts)** \( \int (\cos(x) + \sec(x))^2 \, dx \).
(e) (7 pts) $\int \tan^3(x) \, dx$.

(f) (7 pts) $\int (\ln(x))^2 \, dx$.

(g) (7 pts) $\int \frac{1+x}{x+3e^{-x}} \, dx$.

(h) (7 pts) $\int_0^\infty \frac{x^2}{e^{x^3}} \, dx$
2. (10 pts) Consider the region bounded by the curves $y = 3x^3 - x^2 - 10x$ and $y = -x^2 + 2x$. Set up (but DO NOT EVALUATE) the integral(s) to compute the area of this region.

3. (7 pts) Find $y' = \frac{dy}{dx}$ for $e^{xy} = y^3 + x^2 + \ln(y)$.

4. (8 pts) Consider the region bounded by the graphs $y = \sqrt{2x}$, $y = \frac{x}{2}$, $x = 0$ and $x = 1$. Set up (but DO NOT EVALUATE) the integral to compute the volume of the solid obtained by revolving this region around the $x$-axis.

5. (10 pts) Use the Trapezoidal rule with $n = 4$ to estimate the value of the following integral.

\[ \int_{-1}^{1} \log_{10}(3 + 2x) dx \]

6. (10 pts) The number of wild hogs in a game preserve after $t$ years is given by

\[ N(t) = 500 - \frac{400}{1 + 2t} \]

for $t \geq 0$. What is the average number of wild hogs from $t = 0$ years to $t = 2$ years?

7. Let $f(x) = \frac{1}{4\sqrt{x}}$, $1 \leq x \leq 9$ be the probability density function for a continuous random variable measuring the number of hours in a week that Davis commuters spend driving their automobiles.

(a) (5 pts) Verify that $f(x)$ is a probability density function.

(b) (3 pts) Compute the probability that $x$ takes values greater than or equal to 6.

(c) (6 pts) Compute the expected value $E[x]$.

(d) (6 pts) Compute the median for this random variable.

8. (10 pts) Assume that while chewing Bubble gum, the amount of sugar in the gum decreases following an exponential decay model. If after 1 minute of chewing, 75% of the original mount of sugar is left, what percent of the original amount of sugar will remain after 10 minutes?

9. (10 pts) Suppose that the area bounded under the curve $y = 10e^x$ and $y = 0$ and between $x = 0$ and $x = b$ is equal to 10. What is the value of $b$?
10. (10pts) We are given a probability density function \( f(x) = kx^8 \) for a continuous random variable with the range of values \( 0 \leq x \leq b \). It is known that the median for this random variable is 1. What is the value of \( k \) and \( b \)?