

1. Compute the following integrals (note that some of them are indefinite integrals and some are definite integrals).

(a) (3 pts) $\int (2x + 5)e^{(x^2+5x-1)}dx.$

$$\begin{aligned} \text{Substitute } u &= x^2 + 5x - 1 \Rightarrow du = (2x + 5)dx \\ \Rightarrow \int (2x+5) e^{(x^2+5x-1)} dx &= \int e^u du = e^u + C \\ &= e^{(x^2+5x-1)} + C \end{aligned}$$

(b) (4 pts) $\int_0^1 x(1-x)^{10}dx.$

$$\begin{aligned} \text{Substitute } u &= 1-x \Rightarrow du = -dx \quad \text{and } x = 1-u. \quad \cancel{\text{New limits become}} \\ \Rightarrow \int_0^1 x(1-x)^{10} dx &= \int_0^1 (1-u)u^{10} (-du) \quad \left. \begin{array}{l} \text{New limits become} \\ 0 \text{ becomes } 1 \\ 1 \text{ becomes } 0 \end{array} \right\} \\ &= - \int_1^0 (1-u)u^{10} du = \int_0^1 u^{10}(1-u) du = \left[\frac{u^{11}}{11} - \frac{u^{12}}{12} \right]_0^1 = \boxed{\frac{1}{11} - \frac{1}{12}} \end{aligned}$$

(c) (4 pts) $\int \sec^5(x)\tan(x)dx.$

$$\begin{aligned} \text{Substitute } \sec x &= u \quad du = \sec x \tan x dx \\ \int \sec^5 x \tan x dx &= \int \sec^4 x (\sec x \tan x) dx \\ &= \int u^4 du = \frac{u^5}{5} = \frac{\sec^5 x}{5} + C \end{aligned}$$

(d) (4 pts) $\int \frac{x^4-3}{x^2-1} dx.$

$$(d) \int \frac{x^4 - 3}{x^2 - 1} dx$$

use long division to get

$$\frac{x^4 - 3}{x^2 - 1} = x^2 + 1 + \frac{-2}{x^2 - 1}$$

so

$$\begin{aligned} \int \frac{x^4 - 3}{x^2 - 1} dx &= \int \left(x^2 + 1 + \frac{-2}{x^2 - 1} \right) dx \\ &= \int x^2 dx + \int 1 dx + \int \frac{-2}{x^2 - 1} dx \end{aligned}$$

Compute $\int \frac{-2}{x^2 - 1} dx$ using partial fractions.

$$\frac{-2}{x^2 - 1} = \frac{-2}{(x-1)(x+1)} = \frac{1}{x+1} - \frac{1}{x-1}$$

$$\begin{aligned} \text{so } \int \frac{x^4 - 3}{x^2 - 1} dx &= \int x^2 dx + \int 1 dx + \int \left(\frac{1}{x+1} - \frac{1}{x-1} \right) dx \\ &= \frac{x^3}{3} + x + \int \frac{1}{x+1} dx - \int \frac{1}{x-1} dx \\ &= \frac{x^3}{3} + x + \ln|x+1| - \ln|x-1| + C \end{aligned}$$

(e) (6 pts) $\int \ln(x) dx$. Use integration by parts

$$\int \ln(x) dx = \int(1) \ln(x) dx = \int (\ln x) \cdot (1) dx = \ln(x) \times -\int \frac{1}{x} dx$$

"LATE" rule
 says $f(x) = \ln(x)$
 $g(x) = 1$

$$= \ln(x) \times - \int 1 dx$$

$$= \ln(x)x - x + C$$

(f) (6 pts) $\int x^2 \sin(x) dx$. Use integration by parts.

LATE rule says $f(x) = x^2$, $g(x) = \sin(x)$ $G(x) = \int g(x) dx$

$$\int x^2 \sin(x) dx = x^2 \cos(x) - \int 2x(-\cos x) dx$$

Use by parts again : $f(x) = 2x$ $g(x) = \cos x$

$$= \int \sin x dx$$

$$= -\cos x$$

$$\int 2x \cos x dx = 2x \sin x - \int 2 \sin x dx = 2x \sin x - 2(-\cos x)$$

$$\text{so } \int x^2 \sin x dx = -x^2 \cos x + 2x \sin x + 2 \cos x$$

(g) (6 pts) $\int \sec^4(x) dx$. (Hint: $1 + \tan^2(x) = \sec^2(x)$)

Substitute $u = \tan x$ $du = \sec^2 x dx$

$$\int \sec^4 x dx = \int \sec^2 x \sec^2 x dx = \int (1 + \tan^2 x) \sec^2 x dx$$

$$= \int (1 + u^2) du = u + \frac{u^3}{3}$$

$$= \tan x + \frac{\tan^3 x}{3}$$

(h) (2 pts) Suppose $\int_0^1 \sin(x^2) dx = C$. What is $\int_{-1}^0 \sin(x^2) dx$ in terms of C ?

h) $\sin(x^2)$ is an EVEN function :

$$f(-x) = \sin((-x)^2) = \sin(x^2) = f(x)$$

So $\int_{-1}^1 \sin(x^2) dx = 2 \int_0^1 \sin(x^2) dx = 2C$

~~Also~~ $\int_{-1}^1 \sin(x^2) dx = \int_{-1}^0 \sin(x^2) dx + \int_0^1 \sin(x^2) dx$

~~So~~ So $2C = \int_{-1}^1 \sin(x^2) dx + C$

$$\Rightarrow \boxed{\int_{-1}^1 \sin(x^2) dx = C}$$

$$(i) (5 \text{ pts}) \int e^{2x} \sqrt{1+e^x} dx. \quad \text{Substitute } e^x + 1 = u$$

$$du = e^x dx$$

$$\int e^{2x} \sqrt{1+e^x} dx = \int e^x (\sqrt{1+e^x}) e^x dx = \int (u-1) \sqrt{u} du$$

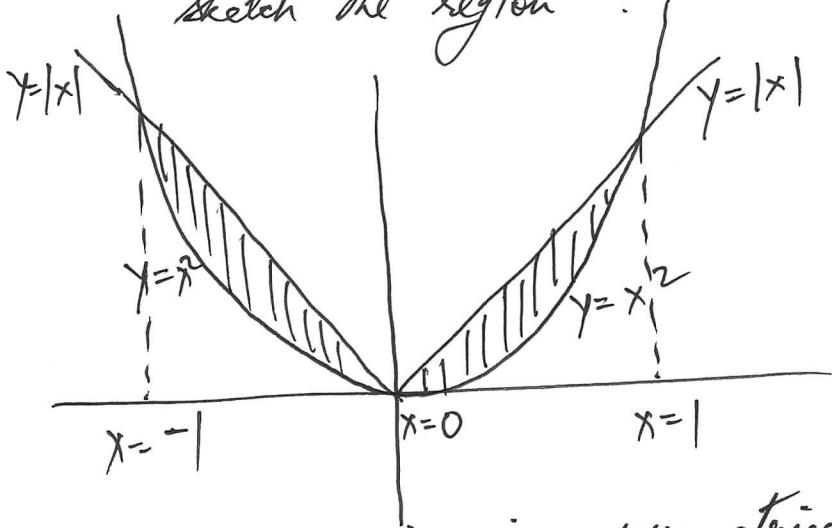
$$= \int u^{\frac{3}{2}} - u^{\frac{1}{2}} du$$

$$= \frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}}$$

$$\left[\frac{(e^x+1)^{\frac{5}{2}}}{\frac{5}{2}} - \frac{(e^x+1)^{\frac{3}{2}}}{\frac{3}{2}} \right]$$

2. (15 pts) Find the area of the region bounded by the graphs of $y = |x|$ and $y = x^2$.

Sketch the region :



since the region is symmetrical about the y-axis.
Area is twice the area from $x=0$ to $x=1$

This area =

$$\int_0^1 (x - x^2) dx$$

$\left[\begin{array}{l} \text{since } y = |x| = x \\ \text{is above } x^2 \\ \text{in } [0, 1] \end{array} \right]$

$$= \left. \frac{x^2}{2} - \frac{x^3}{3} \right|_0^1 = \frac{1}{2} - \frac{1}{3} = \boxed{\frac{1}{6}}$$

so total area = $2 \times \frac{1}{6} = \boxed{\frac{1}{3}}$

3. (10 pts) Consider the region bounded by the curves $y = x^3 - 3x^2 + 3x$ and $y = x^2$. Set up (but DO NOT EVALUATE) the integral(s) to compute the area of this region.

1. Find intersection pts. $x^3 - 3x^2 + 3x = x^2$
 $\Rightarrow x(x^2 - 4x + 3) = 0$
 $\Rightarrow x(x-3)(x-1) = 0 \Rightarrow x=0, 1, 3$

2. In $[0, 1]$, $x^3 - 3x^2 + 3x$ is above x^2
in $[1, 3]$, x^2 is above $x^3 - 3x^2 + 3x$

So area = $\int_0^1 ((x^3 - 3x^2 + 3x) - x^2) dx + \int_1^3 x^2 - (x^3 - 3x^2 + 3x) dx$

4. (10 pts) The temperature of an ice cream sandwich as a function of time t ($t = 0$ being when it is removed from the freezer) is given by

$$T(t) = \frac{60t}{1 + 4t^2}$$

for $0 \leq t \leq 1/2$. What is the average temperature over this time period?

$$\begin{aligned} \text{Average temp} &= \frac{1}{b-a} \int_a^b T(t) dt \\ &= \left(\frac{1}{\frac{1}{2} - 0} \right) \int_0^{\frac{1}{2}} \frac{60t}{1+4t^2} dt \end{aligned}$$

Use substitution $u = 1+4t^2 \Rightarrow du = 8t dt$

$$\Rightarrow \int \frac{60t}{1+4t^2} dt = \int \frac{60}{8} \frac{1}{u} du = \frac{60}{8} \ln|u| = \frac{60}{8} \ln|1+4t^2|$$

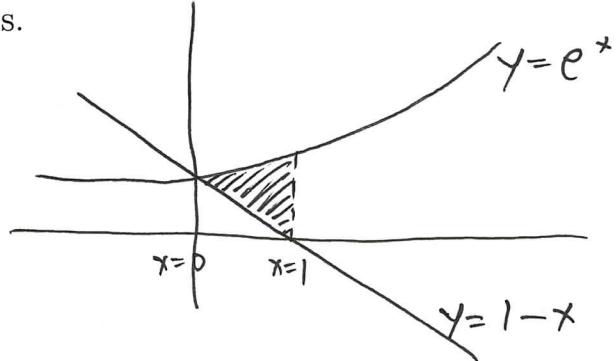
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so avg temp

$$= \frac{1}{\frac{1}{2} - 0} \left[\frac{60}{8} \ln \left(1 + 4t^2 \right) \right]_0^{\frac{1}{2}}$$

$$= \boxed{2 \cdot \frac{60}{8} \ln 2}$$

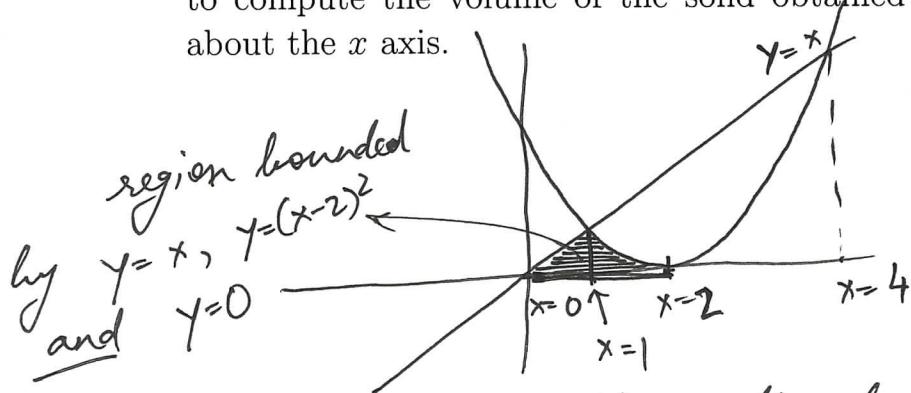
5. (10 pts) Consider the region bounded by the graphs $y = e^x$, $y = 1 - x$, $x = 0$ and $x = 1$. Set up (but DO NOT EVALUATE) the integral to compute the volume of the solid obtained by revolving this region around the x -axis.



e^x is above $1-x$ in the region. so
 volume = $\pi \int_0^1 [(e^x)^2 - (1-x)^2] dx$

using the washer method.

6. (15 pts) Consider the region bounded by the graphs of $y = x$, $y = (x-2)^2$, and $y = 0$. Set up (but DO NOT EVALUATE) the integral(s) to compute the volume of the solid obtained by revolving this region about the x axis.



To get intersection pt
 set $x = (x-2)^2$
 $\Rightarrow x^2 - 5x + 4 = 0$
 ~~$x = 1, 4$~~

Have to break integral into 2 parts
 because different functions are above in different intervals:

$$\pi \int_0^1 (x^2 - 0^2) dx + \pi \int_1^2 ((x-2)^2 - 0^2) dx = \pi \int_0^1 x^2 dx - \pi \int_1^2 (x-2)^2 dx$$